

CONSTRUCTION ELECTRICIAN APPRENTICESHIP PROGRAM

Level 1

Line D: Apply Circuit Concepts



D-2

LEARNING GUIDE D-2 ANALYZE DC CIRCUITS

Foreword

The Industry Training Authority (ITA) is pleased to release this major update of learning resources to support the delivery of the BC Electrician Apprenticeship Program. It was made possible by the dedicated efforts of the Electrical Articulation Committee of BC (EAC).

The EAC is a working group of electrical instructors from institutions across the province and is one of the key stakeholder groups that supports and strengthens industry training in BC. It was the driving force behind the update of the Electrician Apprenticeship Program Learning Guides, supplying the specialized expertise required to incorporate technological, procedural and industry-driven changes. The EAC plays an important role in the province's post-secondary public institutions. As discipline specialists the committee's members share information and engage in discussions of curriculum matters, particularly those affecting student mobility.

ITA would also like to acknowledge the Construction Industry Training Organization (CITO) which provides direction for improving industry training in the construction sector. CITO is responsible for organizing industry and instructor representatives within BC to consult and provide changes related to the BC Construction Electrician Training Program.

We are grateful to EAC for their contributions to the ongoing development of BC Construction Electrician Training Program Learning Guides (materials whose ownership and copyright are maintained by the Province of British Columbia through ITA).

Industry Training Authority

January 2011

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LEVEL 1, LEARNING GUIDE D-2:

ANALYZE DC CIRCUITS

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Learning Objectives

- The learner will be able to describe the operating principles of series circuits.
- The learner will be able to analyze series circuits.
- The learner will be able to describe the operating principles of parallel circuits.
- The learner will be able to analyze parallel circuits.
- The learner will be able to describe the operating principles of combination circuits.
- The learner will be able to analyze combination circuits.
- The learner will be able to describe the operating principles of voltage dividers.
- The learner will be able to analyze voltage dividers.
- The learner will be able to describe the operating principles of bridge circuits.
- The learner will be able to analyze bridge circuits.
- The learner will be able to describe the operating principles of three-wire circuits.
- The learner will be able to analyze three-wire circuits

Activities

- Read and study the topics of Learning Guide D-2: Analyze DC Circuits.
- Complete Self-Tests 1 through 14. Check your answers with the Answer Key located at the end of this Learning Guide.

Resources

All the resources you need are included in this Learning Guide.

BC Trades Modules
www.bctradesmodules.ca

We want your feedback! Please go the BC Trades Modules website to enter comments about specific section(s) that require correction or modification. All submissions will be reviewed and considered for inclusion in the next revision.

SAFETY ADVISORY

Be advised that references to the Workers' Compensation Board of British Columbia safety regulations contained within these materials do not/may not reflect the most recent Occupational Health and Safety Regulation.

The current Standards and Regulation in BC can be obtained at the following website: <http://www.worksafebc.com>.

Please note that it is always the responsibility of any person using these materials to inform him/herself about the Occupational Health and Safety Regulation pertaining to his/her area of work.

Industry Training Authority
January 2011

Learning Task 1:

Describe the characteristics of a series circuit

Electrical components can be connected in different configurations to form circuits for current flow. The simplest of these circuit connections is the *series connection*.

Construction of a series circuit

A series circuit is constructed by connecting all of the circuit components in line with one another. The schematic diagram in Figure 1 is an example of a simple series circuit. In this case, a battery (source) is connected through a switch to three resistors (load devices), all of which are in line with one another.

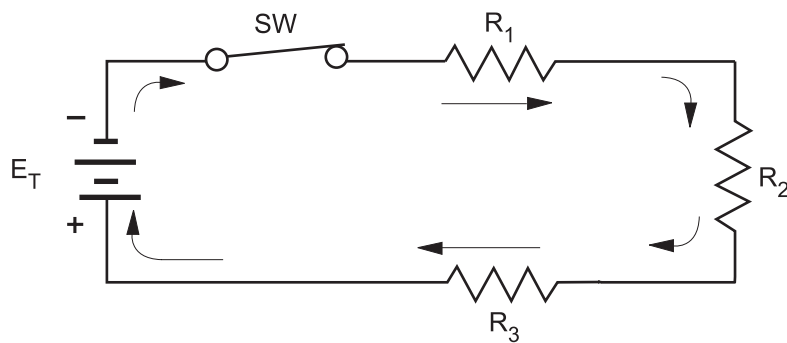


Figure 1—A simple series circuit

Note that when the switch is closed, there is *only one path* for current flow: from the source, through the load devices, and back to the source. Any circuit that provides only one path for current flow is categorized as a series circuit.

If any part of a series circuit is opened, current cannot flow and none of the components will operate. The circuit may be opened by opening the switch or by the failure of a component in the circuit. You have probably encountered strings of Christmas tree lights that are connected as a series circuit. If one lamp burns out (or opens), all the other lamps go out. You are left with the problem of testing each lamp individually to find the failed bulb.

Polarity in a series circuit

Polarity of a circuit, or device, simply identifies which side is negative and which side is positive. It is especially important to identify polarity (or direction of current flow) when you use DC meters such as voltmeters or ammeters. Otherwise the instruments may be damaged.

A DC source, such as a battery, is marked negative on one terminal and positive on the other. If you connect a load to the battery, electrons leave the negative terminal, flow through the load and re-enter the positive terminal of the battery to complete the current loop.

Figure 2 shows a series circuit consisting of five resistors connected to a battery. Notice that there is only one path for the flow of current. The same quantity of electrons that leave the negative terminal of the battery will re-enter the positive terminal after passing through each of the five resistors in succession.

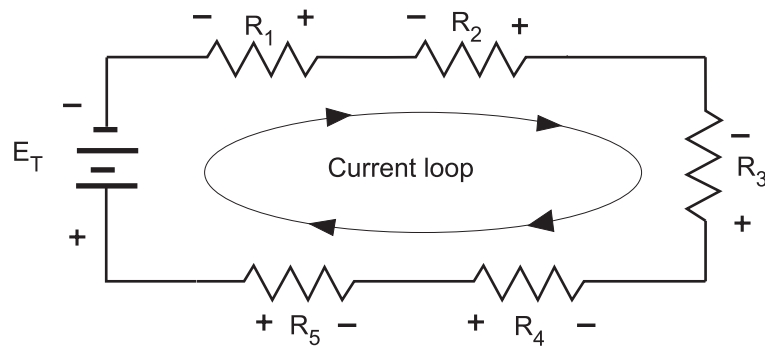


Figure 2—Marking of polarity in a series circuit

As current flows through each resistor, a difference in potential (voltage) is established across each resistor as it offers opposition to current flow. The polarity of the voltage across each resistor is such that the terminal closest to the negative side of the battery is also negative, and the terminal closest to the positive side of the battery is also positive. In other words, the point at which electrons *enter* each resistor is considered *negative*, and the point where electrons *leave* each resistor is considered *positive*. The polarity of the source voltage is reflected by each resistor.

Remember that, viewed as electron flow, current flows from “negative to positive” through any load device.

Where a difference of potential exists, polarity at one point of a circuit is always expressed relative to another point. In Figure 2, the left-hand side of resistor R_1 is negative with respect to the right-hand side and a difference in potential exists between these two points. You will notice that the positive side of R_1 and the negative side of R_2 are really the same point (with no value of resistance in between), so there is no difference in potential (or voltage) between these points.

Three laws for series circuits

There are three fundamental relationships concerning resistance, current and voltage for *all* series circuits. It is important that you learn the three fundamental laws for series circuits so that you will be able to solve more complicated circuit problems.

Resistance

Whenever individual resistances are connected in series, they have the same effect as one large combined resistance. Since there is only one path for current flow in a series circuit, and since each of the resistors is in-line to act as an opposition to this current flow, the overall resistance is the combined opposition of all the in-line resistances.

The total resistance of a series circuit is equal to the sum of all of the individual resistances in the circuit.

This relationship for resistances in a series circuit is expressed as:

$$R_T = R_1 + R_2 + R_3 \dots$$

Using this formula, you find that the total resistance of the circuit in Figure 3 is:

$$R_T = 15\ \Omega + 5\ \Omega + 20\ \Omega = 40\ \Omega$$

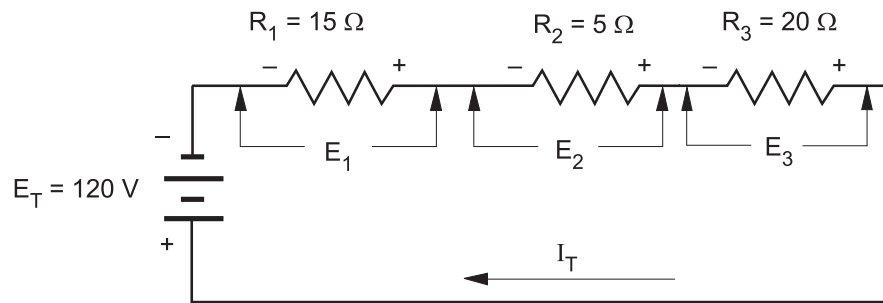


Figure 3—A series circuit

Current

Since there is only one path for electron flow in a series circuit, the current is the same magnitude at any point in the circuit.

The total current in a series circuit is the same as the current through any resistance of the circuit.

This relationship for current in a series circuit is expressed by the following formula:

$$I_T = I_1 = I_2 = I_3 = I_n$$

Given 120 V as the total voltage of the circuit in Figure 3, and having determined the total resistance of the circuit as $40\ \Omega$, you can now apply Ohm's law to determine the total current in this circuit:

$$I_T = \frac{E_T}{R_T} = \frac{120\text{ V}}{40\ \Omega} = 3\text{ A}$$

This total circuit current, $I_T = 3$ amperes, is the same as the current (labelled E_1) through R_1 ; which is the same as the current (E_2) through R_2 ; which is the same as the current (E_3) through R_3 . In other words, the current measured at all sections of the circuit is the same value.

Voltage

You will recall from Ohm's law that, before any current will flow through a resistance, there must be a potential difference, or voltage, available. When resistors are connected in series, they must "share" the total voltage of the source.

The total voltage in a series circuit is equal to the sum of all of the individual voltage drops in the circuit.

As current passes through each resistor in a series circuit, it establishes a difference in potential across each individual resistance. This difference in potential across each resistance is commonly called a *voltage drop*, and its magnitude is in direct proportion to the value of the resistance. The

greater the value of a particular resistance in a series circuit, the higher the voltage drop across that resistor. However, all of the individual voltage drops in a circuit must add up to the value of the source (total applied) voltage.

This relationship of voltages in a series circuit is expressed by the following formula:

$$E_T = E_1 + E_2 + E_3 \dots$$

In the preceding circuit, you can apply Ohm's law as $E = I \times R$ to determine the voltage drop across each individual resistance.

The voltage drop across resistor R_1 is:

$$E_1 = I_1 \times R_1 = 3 \text{ A} \times 15 \Omega = 45 \text{ volts}$$

The voltage drop across resistor R_2 is:

$$E_2 = I_2 \times R_2 = 3 \text{ A} \times 5 \Omega = 15 \text{ volts}$$

The voltage drop across resistor R_3 is:

$$E_3 = I_3 \times R_3 = 3 \text{ A} \times 20 \Omega = 60 \text{ volts}$$

Note that we use the term *voltage drop* because not all of the voltage supplied by the source appears across each resistor.

For this series circuit, the total source voltage is equal to the sum of the individual voltage drops, as confirmed by:

$$E_T = E_1 + E_2 + E_3 = 45 \text{ V} + 15 \text{ V} + 60 \text{ V} = 120 \text{ volts}$$

An open in a series circuit

Refer to Figure 4. If an open is introduced between resistors R_1 and R_2 (for example, by disconnecting a lead), current flow through the circuit is, of course, interrupted. If there is no current flow, the voltage drop across each of the resistive elements is zero (since $E = I \times R$). However, the potential difference of the source appears across the open. If a voltmeter is connected across the open, the reading is the same as if it were connected directly across the terminals of the supply source.

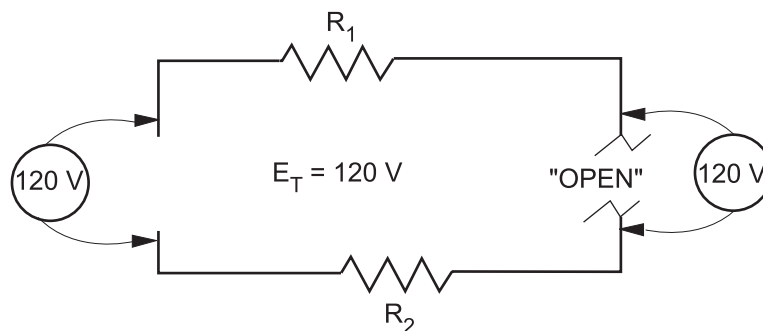


Figure 4—Voltage across an open

In a series lighting circuit, you could quite easily determine which lamp was burnt open simply by measuring the voltage across the lamp-holder terminals, in succession, until you have measured the total source voltage.



Caution! Since the source voltage appears across the open in a series circuit, this represents a *shock hazard*. Be careful not to touch the live parts of the circuit!

Similarly, if a switch is opened, the full source voltage will appear across the switch contacts (the open), even though the voltage across the load devices may be zero.

Application of series circuits

Electrical components or devices are generally connected in series whenever it is necessary to:

- control the amount of current flow in a circuit
- divide the total voltage of a supply

For example:

- Switches are connected in series with loads so that we can energize or de-energize different loads in power distribution systems.
- Protective devices such as fuses and overload relays are connected in series with line conductors so that we can protect circuits against higher than normal currents.
- Variable resistors (called rheostats) are connected in series with loads that may require the amount of current to be varied from time to time. By increasing the resistance of the rheostat, the current can be reduced. By decreasing the resistance of the rheostat, the current can be increased.
- By connecting equal values of resistance in series, the same voltage drop can be obtained across each resistance. Twenty Christmas tree lights connected in series to a 120 V supply would have a voltage rating of 6 V per light.
- Voltage dividers use different values of resistance connected in series to produce a variety of voltage levels from one source of supply voltage.

Disadvantages of series circuits

- An open in any one device will also interrupt current flow to all remaining devices.
- A short in one device will cause an increase in current through all the devices.
- Changing the resistance value of one device will change the current, voltage and power values of all remaining devices.



Now do Self-Test 1 and check your answers.

Self-Test 1

1. How many paths for current flow are there in a series circuit?

2. As more resistors are connected in series, there is an increase in total:

- a. resistance
- b. current
- c. voltage
- d. power

3. A series circuit consists of three resistors. If one resistor is removed, and the remaining two resistors are connected in series to the same source, then the current in each of these two resistors will be:

- a. the same as before
- b. less than before
- c. more than before
- d. zero

4. Two ammeters are connected in series with a circuit, as shown in Figure 1. Which ammeter(s) is/are marked with the correct polarity?

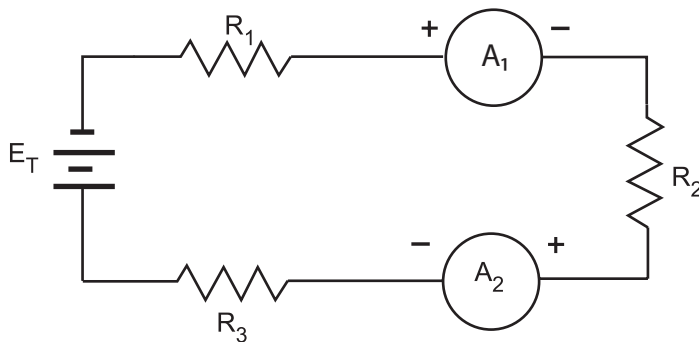


Figure 1—Circuit for Question 4

5. Two resistors are connected in series to a 12 V battery. One resistor is $4\ \Omega$ and the other resistor is $8\ \Omega$. The voltage drop across the $4\ \Omega$ resistor is:
- a. zero volts
 - b. less than the voltage drop across the $8\ \Omega$ resistor
 - c. more than the voltage drop across the $8\ \Omega$ resistor
 - d. the same as the voltage drop across the $8\ \Omega$ resistor
- _____
6. Four equal resistors are connected in series to a 12 V supply. If each resistor has a value of $6\ \Omega$, what is the total circuit resistance?
- _____
7. Three resistors connected in series have a total resistance of 16 ohms. If $R_1 = 4$ ohms and $R_2 = 6$ ohms, what is the value of R_3 ?
- _____
8. A string of 10 Christmas tree lights is connected in series to a 120 V supply. When the circuit is energized, what is the voltage drop across each light?
- _____
9. R_1 and R_2 are in series with a source voltage of 90 V. If E_1 is 30 V, what is the value of E_2 ?
- _____
10. The sum of the IR drops in a series circuit:
- a. is less than the smallest voltage drop
 - b. is usually more than the applied voltage
 - c. equals the average of all of the voltage drops
 - d. equals the applied voltage
- _____
11. When two resistors are connected in series:
- a. There is only one path for current through both resistors.
 - b. They must both have the same resistance values.
 - c. The voltage across each must be the same.
 - d. All of the above are correct.
- _____

12. One of the main disadvantages of a series circuit is that:
- a. An open in the circuit causes voltage to increase across all the components.
 - b. An open in the circuit interrupts current to all the devices.
 - c. A short in one device reduces current through all the remaining devices.
 - d. A short in one device interrupts current to all the devices.
- _____
13. In the circuit in Figure 2, the voltmeter would read:
- a. 0 V
 - b. 30 V
 - c. 60 V
 - d. 120 V
- _____

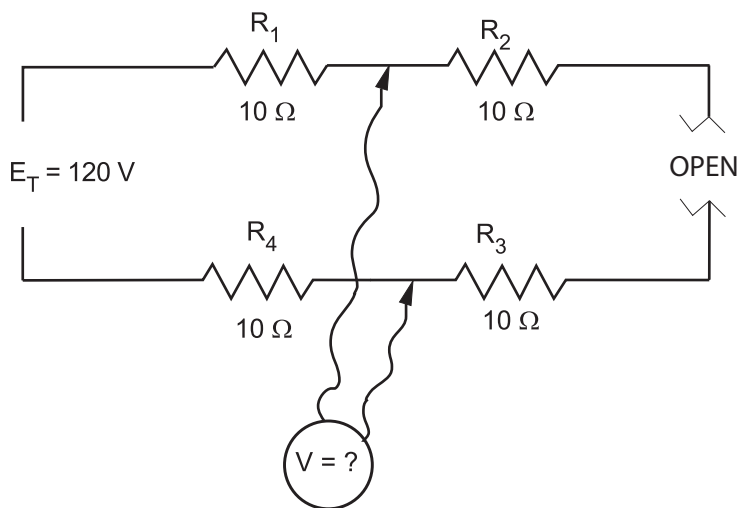


Figure 2—Circuit for Question 13

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 2:

Solve series circuit problems

In your work with series circuits, you must be able to solve problems involving complex calculations dealing with resistance, voltage and current. This requires that you use Ohm's law in different forms:

$$I = \frac{E}{R} \quad \text{and} \quad E = I \times R \quad \text{and} \quad R = \frac{E}{I}$$

and the three fundamental laws of series circuits:

$$I_T = I_1 = I_2 = I_3 \dots$$

$$R_T = R_1 + R_2 + R_3 \dots$$

$$E_T = E_1 + E_2 + E_3 \dots$$

In addition, you must be able to apply fundamental concepts dealing with power and energy.

Power in series circuits

You should recall that power is the rate of using electrical energy, and that power can be calculated by using one of the following three formulas:

$$P = E \times I \quad \text{and} \quad P = I^2 \times R \quad \text{and} \quad P = \frac{E^2}{R}$$

In a series circuit, you can use any of these power equations to find the power dissipated by individual resistive components, or to find the total circuit power. Consider the simple series circuit in Figure 1.

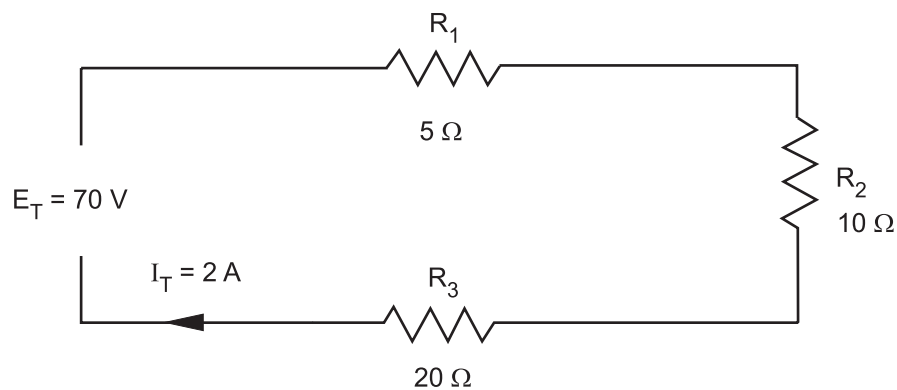


Figure 1—Simple series circuit

From the values listed in the diagram, find the total circuit power by multiplying the total circuit current by the total supply voltage.

$$P_T = E_T \times I_T = 70 \text{ V} \times 2 \text{ A} = 140 \text{ W}$$

Similarly, solve for the power rating of each individual resistance by using $P = I^2 \times R$. Remember, the current through each resistor in a series circuit is the same value as the total circuit current.

$$P_1 = I^2 \times R_1 = (2 \text{ A})^2 \times 5 \Omega = 20 \text{ W}$$

$$P_2 = I^2 \times R_2 = (2 \text{ A})^2 \times 10 \Omega = 40 \text{ W}$$

$$P_3 = I^2 \times R_3 = (2 \text{ A})^2 \times 20 \Omega = \underline{80 \text{ W}}$$

$$P_T = 140 \text{ W}$$

By adding the individual power ratings, you can see that the resultant value of power is the same as the total power we calculated previously by using total circuit values.

In a series circuit, the total power developed is equal to the sum of the power values of the individual resistance components.

This relationship for power in a series circuit is expressed by the following formula:

$$P_T = P_1 + P_2 + P_3 \dots$$

Sample problems

You should now be ready to solve problems dealing with series circuits. Review the solutions for the following problems.

Example 1: For the circuit shown in Figure 2, solve for the unknown value of resistance R_3 and its power developed.

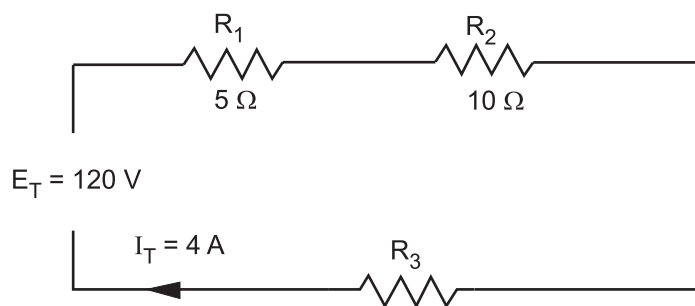


Figure 2—Series circuit for Example 1

Solution:

From the values given, you can solve for the total circuit resistance and then subtract the known values of resistance to find the value of R_3 . Applying Ohm's law, you find the total resistance is:

$$R_T = \frac{E_T}{I_T} = \frac{120 \text{ V}}{4 \text{ A}} = 30 \Omega$$

To solve for R_3 insert the known values into the resistance formula for a series circuit by using:

$$R_T = R_1 + R_2 + R_3$$

$$R_3 = R_T - R_1 - R_2$$

$$R_3 = 30 - 5 - 10 = 15 \Omega$$

The quickest way to solve for the power in R_3 is:

$$P_3 = I^2 \times R_3 = (4 \text{ A})^2 \times 15 \Omega = 240 \text{ W}$$

Example 2: Solve for the unknown values of source voltage, line current and resistance of R_3 for the circuit shown in Figure 3.

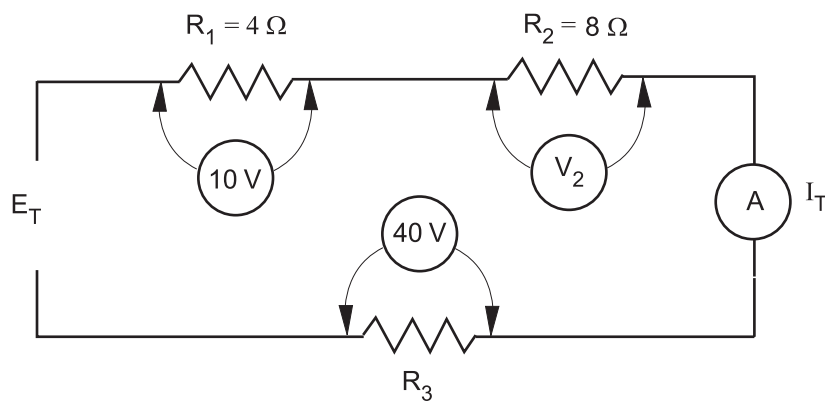


Figure 3—Series circuit for Example 2

Solution:

Using the values associated with R_1 , solve for the current I_1 , which is the same value as I_T .

$$I_T = I_1 = \frac{E_1}{R_1} = \frac{10 \text{ V}}{4 \Omega} = 2.5 \text{ A}$$

The value of R_3 can now be found, since $I_T = I_1 = I_3$.

$$R_3 = \frac{E_3}{I_3} = \frac{40 \text{ V}}{2.5 \text{ A}} = 16 \Omega$$

The total supply voltage can now be calculated by using Ohm's law with the total resistance and total current values. First find R_T , then E_T .

$$\begin{aligned} R_T &= R_1 + R_2 + R_3 \\ &= 4 \Omega + 8 \Omega + 16 \Omega = 28 \Omega \end{aligned}$$

$$\begin{aligned} E_T &= I_T \times R_T \\ &= 2.5 \text{ A} \times 28 \Omega = 70 \text{ V} \end{aligned}$$

Another way to find the total voltage is to find the voltage drop E_2 , and then add all of the individual voltage drops. By this method:

$$\begin{aligned} E_2 &= I_2 \times R_2 \\ &= 2.5 \text{ A} \times 8 \Omega = 20 \text{ V} \end{aligned}$$

$$\begin{aligned} E_T &= E_1 + E_2 + E_3 \\ &= 10 \text{ V} + 20 \text{ V} + 40 \text{ V} = 70 \text{ V} \end{aligned}$$

Proportional voltage drops

In series circuits, the total voltage is the sum of the individual voltage drops in the circuit, and the relationship $E = I \times R$ is used to calculate the voltage drops across each resistor. Since the current is the same through each resistor, the voltage drop across each resistor is directly proportional to the value of resistance. In other words, the greater the value of a resistor in a series circuit, the higher the voltage drop. Consider the simple series circuit in Figure 4.

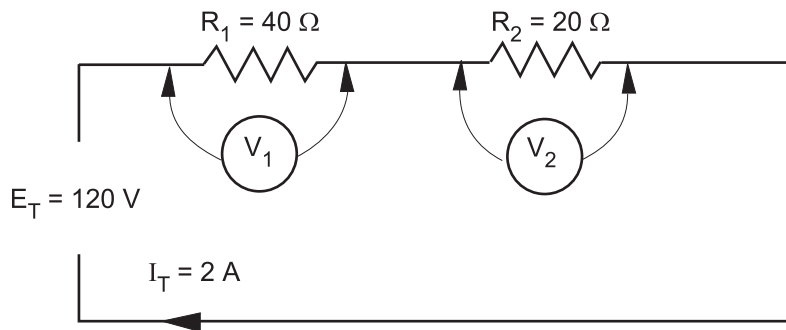


Figure 4—Series circuit

From the values given above, you can easily calculate the voltage drops across each resistor by:

$$E_1 = I_1 \times R_1 = 2 \text{ A} \times 40 \Omega = 80 \text{ V}$$

$$E_2 = I_2 \times R_2 = 2 \text{ A} \times 20 \Omega = 40 \text{ V}$$

The voltage drop of 80 V across the 40Ω resistor is twice the voltage drop across the 20Ω resistor. Since the current is the same through both resistors, the voltage drops are directly proportional to the resistance (2:1). This relationship can be expressed as follows:

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}$$

This proportional relationship (ratio) is very useful for quickly solving for resistance or voltage drop.

Example 3: Four resistors are connected in series to a 120 V supply, as shown in Figure 5. If the voltage drop across R_2 is 24 V, determine the voltage drop across each of the remaining three resistors.

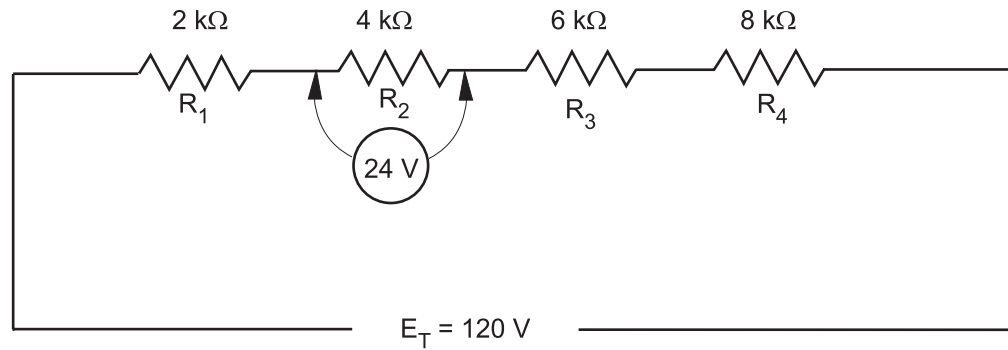


Figure 5—Series circuit for Example 3

Solution:

You could first find the total resistance in this series circuit; then calculate the current by Ohm's law; and finally calculate the individual voltage drops. But the circuit can quickly be solved by using the ratio technique discussed.

The ratio between R_1 and R_2 is:

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}$$

Substitute the known values and solve for E_1 :

$$\frac{E_1}{24\text{ V}} = \frac{2\text{ k}\Omega}{4\text{ k}\Omega} \text{ Therefore } E_1 = 12\text{ V}$$

Similarly, we can establish a ratio to solve for E_3 by using:

$$\begin{aligned} \frac{E_2}{E_3} &= \frac{R_2}{R_3} \\ \frac{24\text{ V}}{E_3} &= \frac{4\text{ k}\Omega}{6\text{ k}\Omega} \\ \therefore E_3 &= 36\text{ V} \end{aligned}$$

And again for E_4 by using:

$$\begin{aligned} \frac{E_2}{E_4} &= \frac{R_2}{R_4} \\ \frac{24\text{ V}}{E_4} &= \frac{4\text{ k}\Omega}{8\text{ k}\Omega} \\ \therefore E_4 &= 48\text{ V} \end{aligned}$$

Tips for solving series circuits

You will often be given word problems and asked to solve for an unknown quantity (or quantities). In these cases, always draw a *schematic diagram* showing the given circuit information (just as in the previous examples).

Often, to solve series circuit problems, it is most useful to find the information that will allow you to calculate a current value. This current can then be used throughout the entire circuit.

Example 4: Three resistors are connected in series to a 12 V battery. One resistor has a value of $10\ \Omega$ with a voltage drop of 2 V. The other two resistors have voltage drops of 6 V and 4 V, respectively. What are the resistance and power values of the two unknown resistors?

Solution:

First, draw a labelled schematic from the information given.

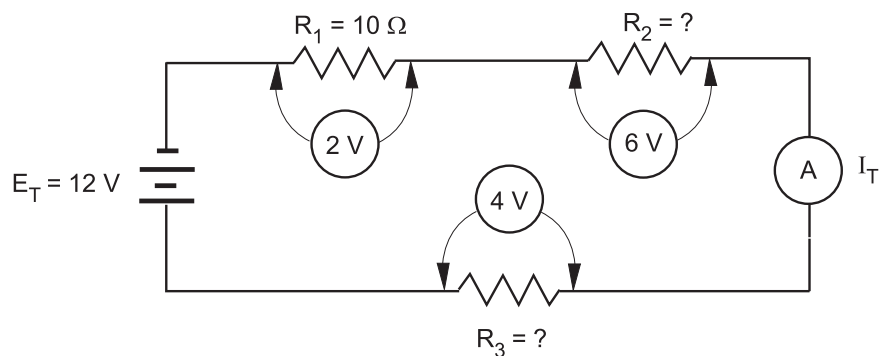


Figure 6—Schematic for solving Example 4

Calculate the total current from the values of R_1 above.

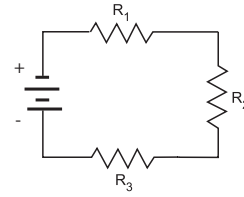
$$I_T = I_1 = \frac{E_1}{R_1} = \frac{2\text{ V}}{10\ \Omega} = 0.2\text{ A}$$

Then, because you know that the current is the same for the other two resistors:

$$R_2 = \frac{E_2}{I_2} = \frac{6\text{ V}}{0.2\text{ A}} = 30\ \Omega \quad \text{and} \quad R_3 = \frac{E_3}{I_3} = \frac{4\text{ V}}{0.2\text{ A}} = 20\ \Omega$$

Notice that you could also use the ratio method to solve for the resistance values. Now apply one of the power equations, such as:

$$\begin{aligned}
 P_2 &= E_2 \times I_2 \\
 &= 6 \text{ V} \times 0.2 \text{ A} \\
 &= 1.2 \text{ W}
 \end{aligned}
 \quad \text{and} \quad
 \begin{aligned}
 P_3 &= E_3 \times I_3 \\
 &= 4 \text{ V} \times 0.2 \text{ A} \\
 &= 0.8 \text{ W}
 \end{aligned}$$



Some series circuit problems require you to solve many different values of voltage, current, resistance and power. In these cases, you may find it handy to systematically list all known and unknown values in a chart. Use a chart like this one in Table 1 to record the individual values of three resistors and the total circuit values.

Table 1 Circuit value chart

	R	E	I	P
R_1				
R_2				
R_3				
Total				

When using this chart method, remember to fill in the given information first. This will make it easier to determine which values can be used for other calculations.

Kirchhoff's voltage law

There are two fundamental laws relating to electric circuits, known as Kirchhoff's law. One deals with voltage, the other with current.

Simply put, Kirchhoff's voltage law says that:

Around any closed loop, the algebraic sum of all the voltages is zero.

This law is used in conjunction with *polarity* based on the direction of current flow in a circuit. If the resistive voltage drops are considered negative quantities, then by comparison, the source voltage is considered a positive quantity (a voltage rise).

Expressing Kirchhoff's law as a formula, you have:

$$+ E_T - E_{R1} - E_{R2} - E_{R3} = 0$$

By transposing, notice that the law is simply rewritten for series circuits as:

$$E_T = E_{R1} + E_{R2} + E_{R3} \dots$$

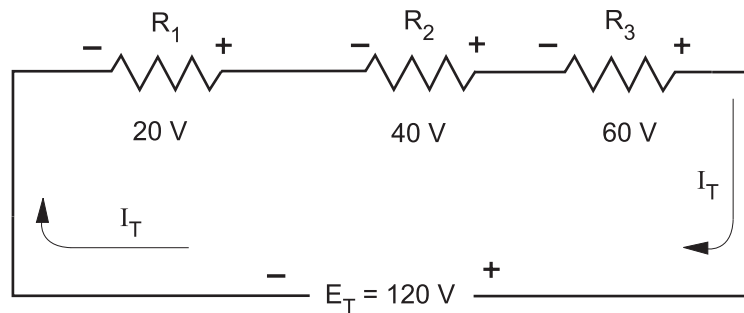


Figure 7—Kirchhoff's voltage law

If you trace the current loop in Figure 7, you find that as you travel from negative to positive through each of the load resistors, you also go from positive to negative in the voltage source. It is *this* direction of current, or polarity, that is assigned to the voltage values when you apply Kirchhoff's law.

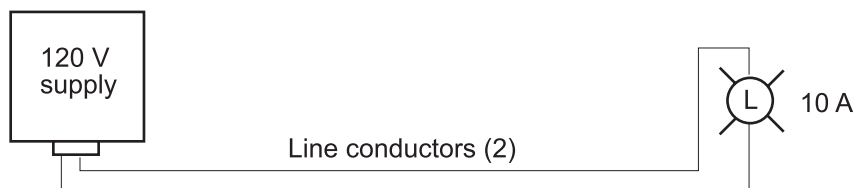
Substituting the values in Figure 7 into Kirchhoff's voltage law gives us the following:

$$+ 120 \text{ V} - 20 \text{ V} - 40 \text{ V} - 60 \text{ V} = 0$$

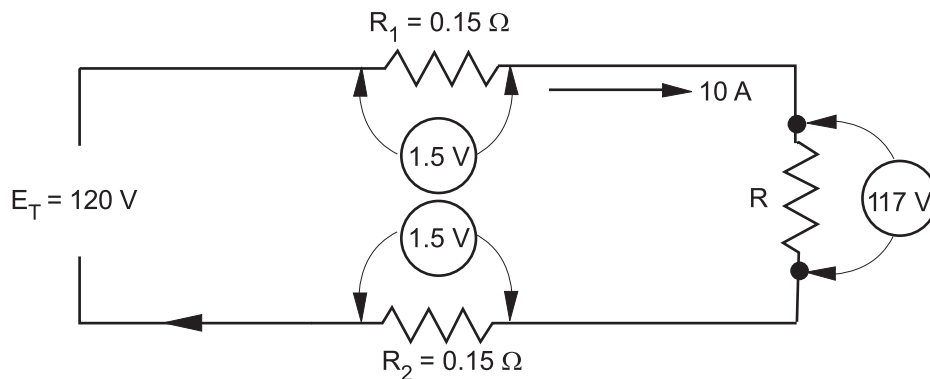
Effects of line drop and line loss

Elements such as copper and aluminum are used as conductors because they offer little opposition to the flow of current. Although the resistance of conductors is often neglected in simple circuit analysis, it may be necessary to consider the resistance of lines in practical applications.

Figure 8(a) illustrates a simple branch circuit with a 120 V source supplying a 10 A load. The two line conductors may be visualized as two resistors with small resistance connected in series with the load resistance.



(a) Simple 120 V branch circuit



(b) Schematic equivalent

Figure 8—Line resistance and voltage drop

Line drop

As the 10 A current flows through each line resistance of $0.15\ \Omega$, a small voltage drop appears across each line. This voltage drop across the line conductors is commonly referred to as a *line drop*, and is calculated as:

$$\begin{aligned} E &= I \times R \\ &= 10\text{ A} \times 0.15\ \Omega \\ &= 1.5\text{ V per line} \end{aligned}$$

Since there are two lines, the total drop is $2 \times 1.5\text{ V} = 3\text{ V}$. The net voltage across the load (117 V) is less than the source voltage (120 V) because of the 3 V line drop.

In some applications it may be necessary to use larger conductors, which have lower resistance, so that the line drop does not reduce the load voltage too significantly.

Line loss

Another term associated with conductors is *line loss*. This is a power loss expressed in watts, and is related to heat energy dissipation as current flows through the resistance of the line conductors. Line loss is calculated by using one of the power equations.

Then, by using the values from Figure 8, the total line loss for the two $0.15\ \Omega$ lines is:

$$\begin{aligned} P &= I^2 \times R \\ &= (10\text{ A})^2 \times 0.30\ \Omega \\ &= 30\text{ W} \end{aligned}$$

Calculations involving line loss are generally only significant when determining the *efficiency* of electrical circuits and equipment.

Do not confuse the terms *line drop* and *line loss*.

- *Line drop* is expressed in volts.
- *Line loss* is expressed in watts.



Now do Self-Test 2 and check your answers.

Self-Test 2

- Three resistors, $R_1 = 3.6 \, \Omega$, $R_2 = 4.5 \, \Omega$ and $R_3 = 6.2 \, \Omega$, are connected in series to a 120 V supply. What is the total current in the circuit?

- A strip heater consists of four equal resistive elements connected in series. When connected to 220 V, the line current is 11 A. What is the resistance of each element?

- Three resistors are connected in series to a 100 V source. If $R_1 = 250 \, \Omega$, $R_2 = 470 \, \Omega$ and $R_3 = 500 \, \Omega$, what are the line current and voltage drops across each resistor?

- For the circuit in Figure 1, calculate the following:
 - E_1 _____
 - E_2 _____
 - E_3 _____
 - R_3 _____
 - R_T _____
 - P_T _____

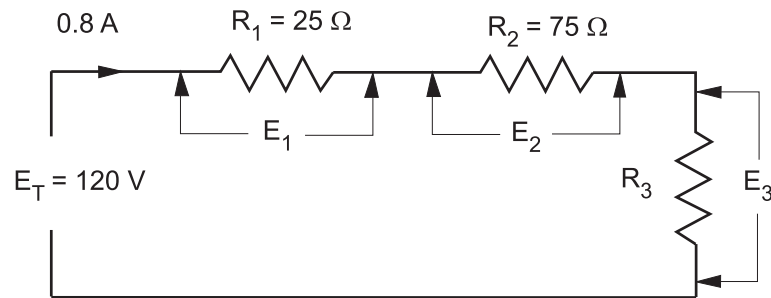


Figure 1—Circuit for Question 4

5. For the circuit in Figure 2, calculate the following:

- a. E_1 _____
- b. E_2 _____
- c. E_3 _____
- d. E_T _____
- e. R_T _____
- f. P_2 _____
- g. P_3 _____
- h. P_T _____

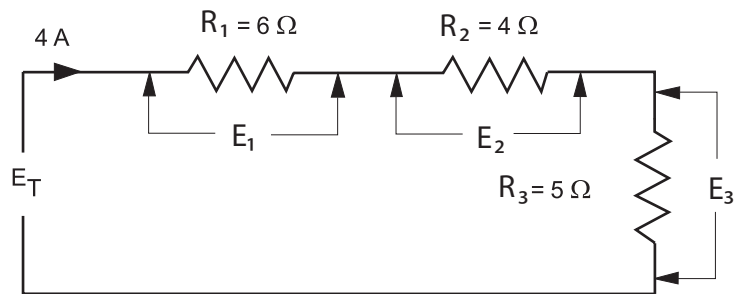


Figure 2—Circuit for Question 5

6. Calculate each of the following values in the circuit of Figure 3.

- a. E_1 _____
- b. E_2 _____
- c. R_2 _____
- d. R_3 _____
- e. R_T _____
- f. P_T _____

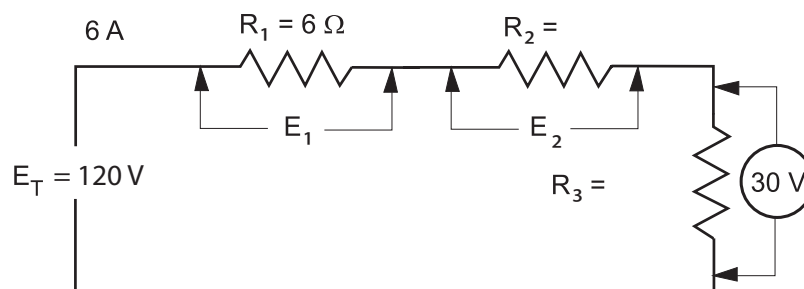


Figure 3—Circuit for Question 6

7. Calculate each of the following values for the circuit of Figure 4.

- a. E_1 _____
- b. E_2 _____
- c. E_T _____
- d. R_3 _____
- e. R_1 _____
- f. P_T _____

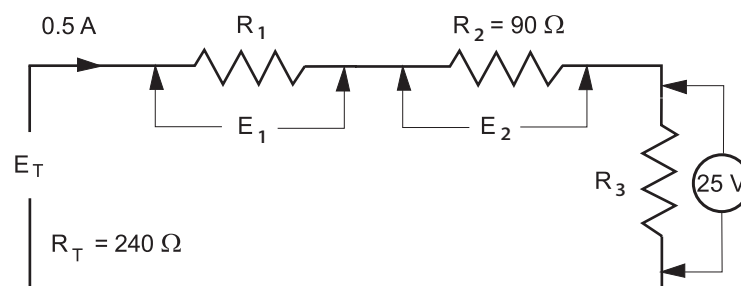


Figure 4—Circuit for Question 7

The circuit shown in Figure 5 uses a variable resistor to control the current to a load. Use this circuit to answer Questions 8 to 11.

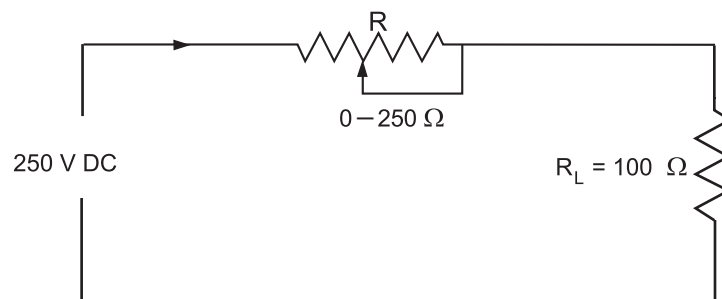


Figure 5—Circuit for Questions 8 to 11

8. How much load current will flow with the slider turned fully to the left?

9. How much current will flow with the slider turned fully to the right?

10. With the circuit current adjusted to 1 ampere, how much resistance is connected in series with the load?

11. A variable resistor applied in this manner is called a:

12. Line loss is measured in:

- a. amps
- b. volts
- c. watts
- d. ohms

13. Line drop is measured in:

- a. amps
- b. volts
- c. watts
- d. ohms

14. When using larger conductors for a given value of current, the line drop:

- a. increases
- b. decreases
- c. is not affected

15. When using smaller conductors for a given value of current, the line loss:

- a. increases
- b. decreases
- c. is not affected

16. From the information given in Figure 6, determine the total:

- a. line drop
- b. line loss

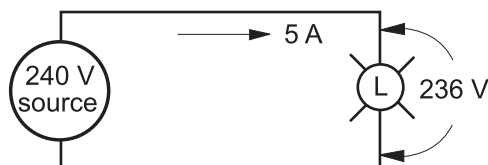


Figure 6—Find totals for Question 16 a and b

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 3:

Describe the effects of voltage sources in series

Voltage sources are sometimes connected in series to produce a higher voltage value. This is common in devices such as flashlights and portable transistor radios, in which battery cells are used. In AC applications, generator coils and transformer windings are often connected in series.

In order to obtain a higher voltage output from series-connected sources, you must observe correct polarity. In Figure 1, a net voltage of 6 V is obtained if the individual 1.5 V emfs are acting in the same direction, or connected *series aiding*.

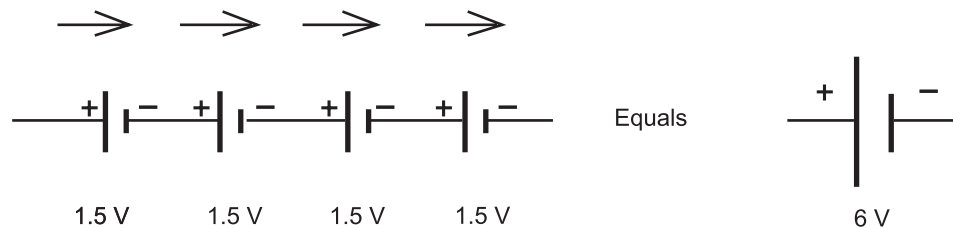


Figure 1—Emfs connected series aiding

For the voltages to accumulate, the negative terminal of one source connects in series with the positive terminal of the next source, and so on. That is, unlike terminals connect. The effect produced is as if you had one large battery of 6 V. Generally, the individual source voltages are of equal value, but they don't necessarily have to be, provided they are capable of conducting the same value of current.

When voltage sources are connected *series opposing*, the net voltage value is derived by subtraction. This is illustrated in Figure 2.

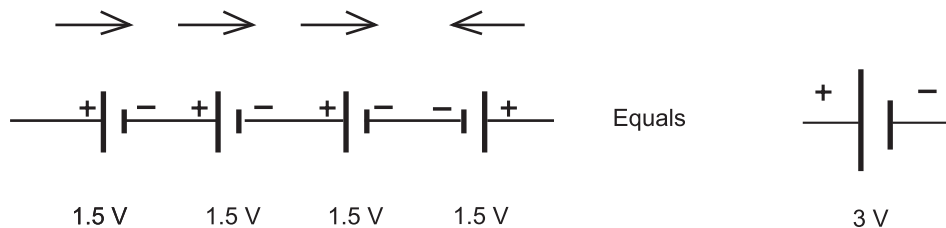


Figure 2—Emfs connected series opposing

- Three of the cells are connected series aiding to produce 4.5 V.
- One cell is connected with opposite polarity of 1.5 V.
- Net voltage is $4.5 \text{ V} - 1.5 \text{ V} = 3 \text{ V}$.
- Overall polarity acts in the direction of the largest emfs.



Now do Self-Test 3 and check your answers.

Self-Test 3

1. If the battery cells in each of the circuits shown in Figures 1, 2 and 3 are rated as 1.5 V, determine the following and write your answers on each figure.
 - the value of voltage expected on each of the voltmeters
 - the proper connected polarity on each voltmeter

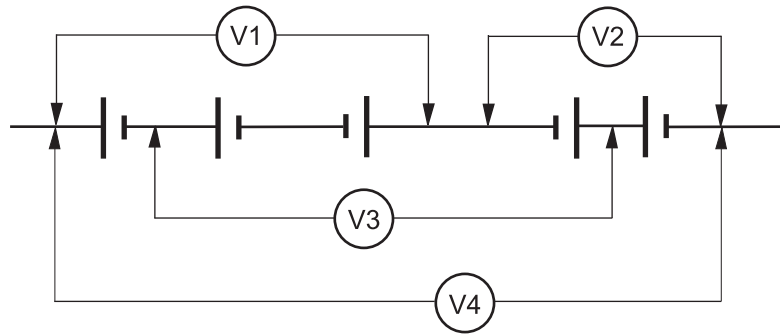


Figure 1—Circuit

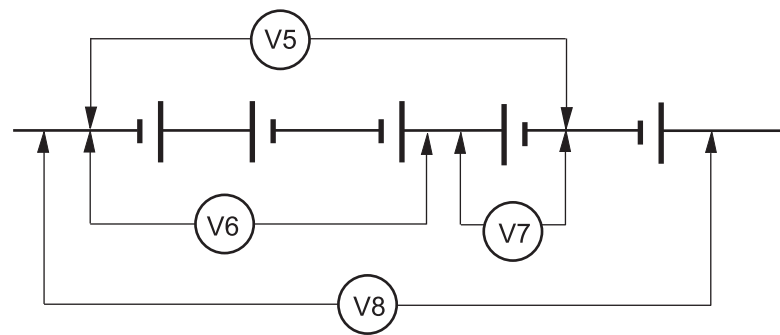


Figure 2—Circuit

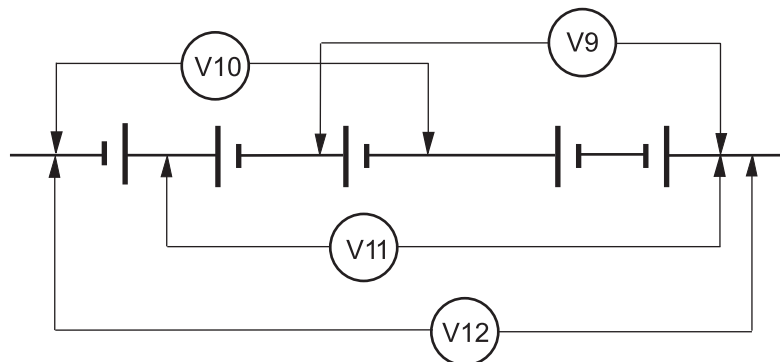


Figure 3—Circuit

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 4:

Describe the characteristics of a parallel circuit

The parallel circuit is probably the most common type of circuit you will encounter. Loads in power distribution systems are mostly connected in parallel with each other in one way or another.

Construction of a parallel circuit

A parallel circuit is constructed by connecting the terminals of all the individual load devices so that the same value of voltage appears across each component. In Figure 1, you can see that each of the three resistors receives the same voltage from the source.

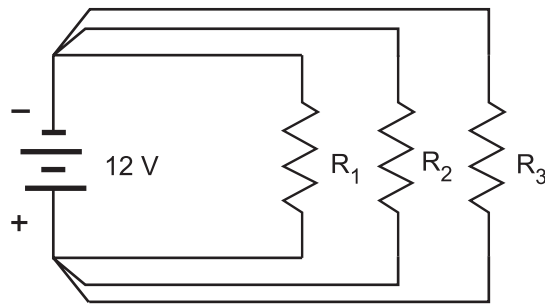


Figure 1—A parallel circuit

Figure 2 shows the more traditional way of representing the same circuit. Note that in this circuit arrangement:

- The total supply voltage appears across each of the three resistors.
- There are three separate paths (or branches) for current flow—each leaving the negative terminal of the supply and returning to the positive terminal.

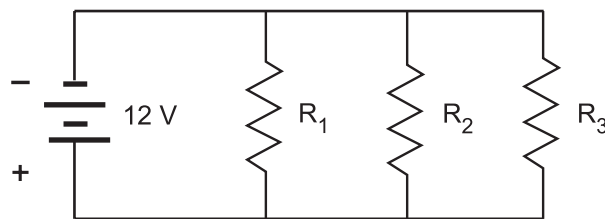


Figure 2—Usual representation of parallel circuit

The two fundamental characteristics of any parallel circuit are that:

- The voltage across each branch is the same, and
- There is more than one path for current to flow through.

In contrast to a series circuit, current still flows to the remaining devices in the circuit if any one branch or component in a parallel circuit is opened.

Polarity in a parallel circuit

Just as in series circuits, electron current flows “from negative to positive” through each of the load components in a parallel circuit.

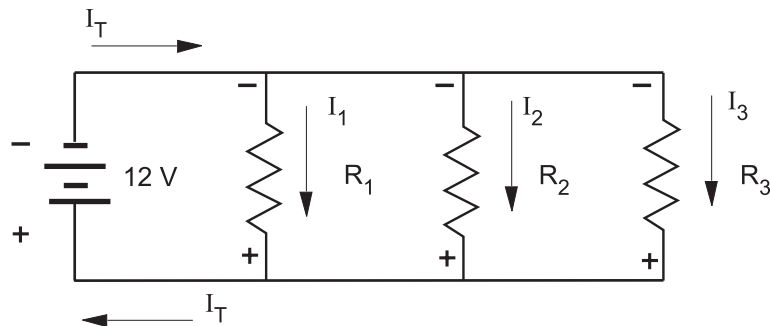


Figure 3—Polarity in a parallel circuit

As illustrated in Figure 3, electrons leave the negative terminal of the source and flow from negative to positive through each of the load resistors.



Note that the polarity of each of the resistors is the same as the polarity of the source.

Polarity is always expressed from one point of a circuit relative to another point with a different electrical potential. Note that in Figure 3 the top side of each resistor, which is marked negative, is in effect the *same point*. No difference in potential exists between any of these *like terminals*.

Also notice that the individual currents through each resistor (I_1 , I_2 , I_3) together constitute the total current (I_T) drawn from the source.

Current and resistance in a parallel circuit

By Ohm's law, you know that the current in a circuit is inversely proportional to the resistance, which means that if the resistance goes down, the current goes up.

As more and more resistors are connected in parallel to a source of supply, there are more paths for current flow and, as a result, more current is drawn by the circuit. This has the effect of reducing the net (total) resistance of the circuit.

Consider the simple parallel circuit shown in Figure 4.

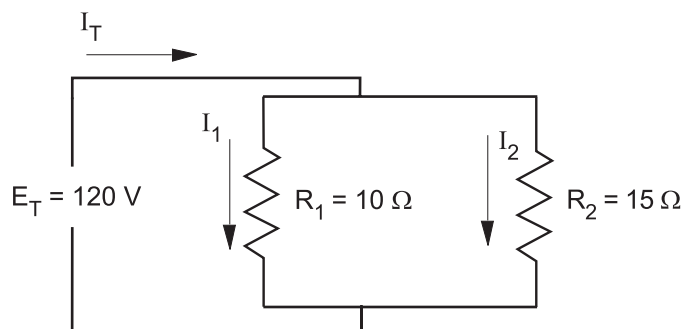


Figure 4—Two resistors in parallel

Using Ohm's law, we can easily determine the current through each resistor:

$$I_1 = \frac{E_T}{R_1} = \frac{120 \text{ V}}{10 \Omega} = 12 \text{ A}$$

and

$$I_2 = \frac{E_T}{R_2} = \frac{120 \text{ V}}{15 \Omega} = 8 \text{ A}$$

In a parallel circuit, the total current is equal to the sum of the individual branch currents. In this case, the total current is calculated as 20 amperes.

$$\begin{aligned} I_T &= I_1 + I_2 \\ &= 12 \text{ A} + 8 \text{ A} \\ &= 20 \text{ A} \end{aligned}$$

The total resistance of this circuit can be found by applying Ohm's law and using the total circuit voltage and total circuit current.

$$R_2 = \frac{E_T}{I_T} = \frac{120 \text{ V}}{20 \text{ A}} = 6 \Omega$$

Notice that the *total* circuit resistance is 6 ohms, which is *less* than either of the individual branch resistance values (R_1 and R_2).

As you connect more resistors in parallel, there are more branches for current flow. This *increases* the total circuit *current* and consequently *decreases* the total circuit *resistance* of the parallel circuit.

Laws of a parallel circuit

There are three fundamental relationships concerning voltage, current and resistance in all parallel circuits.

Voltage

In a parallel circuit, each load resistor acts as an independent branch circuit, and because of this, each branch "sees" the entire voltage of the supply.

Total voltage of a parallel circuit has the same value as the voltage across each branch.

This relationship for voltages in a parallel circuit is expressed as:

$$E_T = E_1 = E_2 = E_3 \dots$$

In the circuit in Figure 5, the voltage across each branch is 120 V.

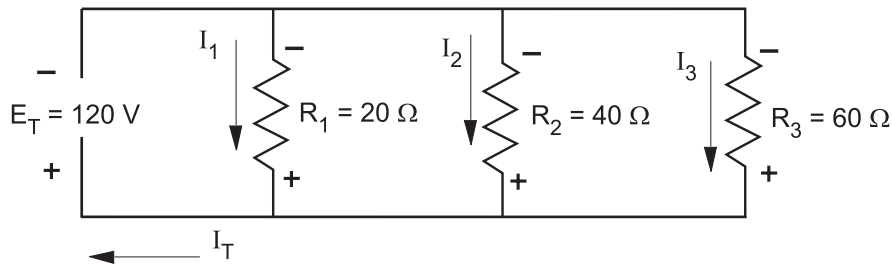


Figure 5—A parallel circuit with three branches

Current

A parallel circuit has more than one path for current flow. The number of current paths is determined by the number of load resistors connected in parallel.

Total current in a parallel circuit is the sum of the individual branch currents.

This relationship for currents in a parallel circuit is expressed as:

$$I_T = I_1 + I_2 + I_3 \dots$$

To solve for the total (line) current in Figure 5, you must first determine the individual branch currents by applying Ohm's law:

$$I_1 = \frac{E_1}{R_1} = \frac{120 \text{ V}}{20 \Omega} = 6 \text{ A}$$

and

$$I_2 = \frac{E_2}{R_2} = \frac{120 \text{ V}}{40 \Omega} = 3 \text{ A}$$

and

$$I_3 = \frac{E_3}{R_3} = \frac{120 \text{ V}}{60 \Omega} = 2 \text{ A}$$

Using the branch currents, you can now solve for the total (line) current as follows:

$$\begin{aligned} I_T &= I_1 + I_2 + I_3 \\ &= 6 \text{ A} + 3 \text{ A} + 2 \text{ A} \\ &= 11 \text{ A} \end{aligned}$$

Resistance

Whenever more resistances are connected in parallel, they have the effect of reducing the overall circuit resistance.

The net resistance of a parallel circuit is always less than any of the individual branch resistance values.

There are several different ways of determining the total resistance of a parallel circuit. One of the most common methods is to use the reciprocal equation:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

For the circuit in Figure 5, calculate the total resistance by substituting appropriate values in this reciprocal equation:

$$\frac{1}{R_T} = \frac{1}{20\ \Omega} + \frac{1}{40\ \Omega} + \frac{1}{60\ \Omega} + \dots$$

Next, find a common denominator (and omit the units while you calculate):

$$\frac{1}{R_T} = \frac{6}{120} + \frac{3}{120} + \frac{2}{120} = \frac{11}{120}$$

Then transpose:

$$R_T = \frac{120}{11} = 10.9\ \Omega$$

Later, you will be shown several different methods of solving for resistance in parallel circuits.

Power in a parallel circuit

The same power equations you used in series circuits apply to parallel circuits. For example, to solve for the power developed by resistor R_1 in Figure 5 on the previous page, use:

$$P_1 = \frac{E_1^2}{R_1} \quad \text{where } E_1 = 120\ \text{V and } R_1 = 20\ \Omega$$

$$\text{or } P_1 = I_1^2 \times R \quad \text{where } I_1 = 6\ \text{A}$$

$$P_1 = E_1 \times I_1 \quad \text{resulting in } P_1 = 720\ \text{watts}$$

In the same way, you can find the power for resistor R_2 ($P_2 = 360\ \text{W}$) and for resistor R_3 ($P_3 = 240\ \text{W}$) by using any of the above formulas with the subscripts suitably changed.

You can find the total circuit power (P_T) in the following two ways:

- Use the total circuit values of voltage, current or resistance in the above formulas.
- Find the sum of the wattage dissipated by the individual branch resistances.

$$P_T = P_1 + P_2 + P_3 \dots$$

Kirchhoff's current law

When you worked with series circuits, you were introduced to Kirchhoff's voltage law. When you work with parallel circuits, you will commonly refer to Kirchhoff's current law.

Simply put, Kirchhoff's current law states:

The sum of the currents entering a junction (node) is equal to the sum of the currents leaving that junction.

Kirchhoff's law is rather like a current conservation law: what goes in must come out. At any junction where circuits branch, the total outgoing amperage must equal the total incoming amperage. This relationship is illustrated in Figure 6.

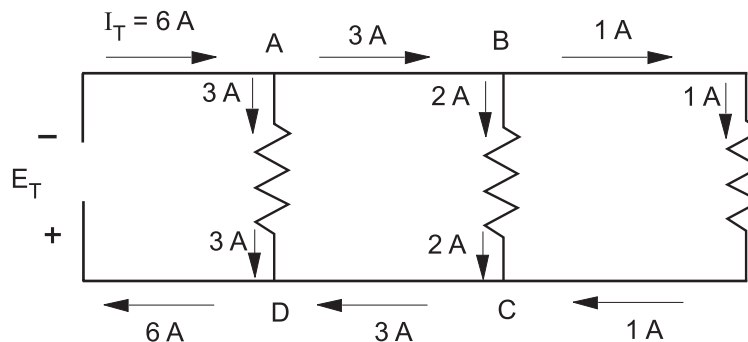


Figure 6—Circuit illustrating Kirchhoff's current law

- At node A, the current divides into two branches. Notice that 6 A enters at node A and that a total of 6 A (3 A + 3 A) leaves node A.
- At node B, a current of 3 A enters and a total current of 3 A exits (2 A + 1 A).
- At node C, 3 A enter (1 A + 2 A) and 3 A leave.
- At node D, 6 A enter (3 A + 3 A) and 6 A leave.

You can see that the total current exiting the circuit at the bottom of Figure 6 is equal to the line current entering at the top of the circuit.

Although this law may seem rather obvious and simple, Kirchhoff's current law is used with Kirchhoff's voltage law to solve complex circuit problems.



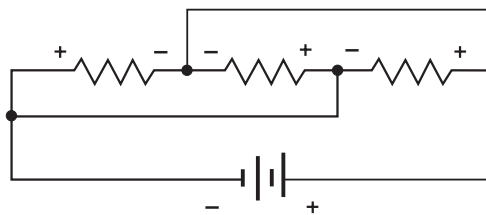
Now do Self-Test 4 and check your answers.

Self-Test 4

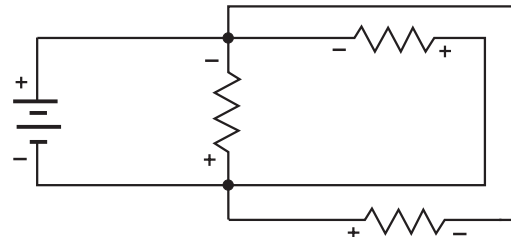
1. A parallel circuit must have at least _____ paths for electrons to flow in.

- a. 0
- b. 1
- c. 2
- d. 3

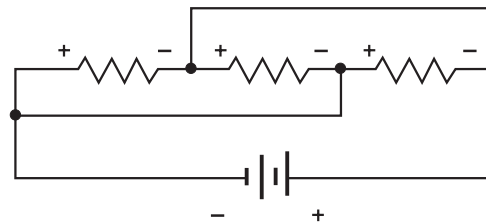
2. Which of the four circuits in Figure 1 is marked with correct load polarity? _____



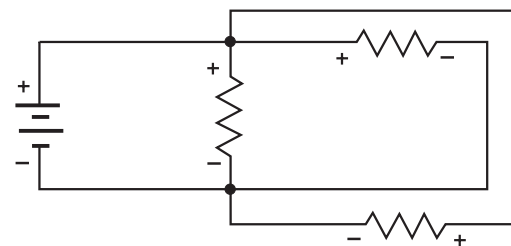
Circuit a



Circuit b



Circuit c



Circuit d

Figure 1—Circuits for Questions 2 and 3

3. Ignoring any incorrect polarity markings, are all the circuits in Figure 1 considered parallel circuits?

4. A $4\ \Omega$ resistor is connected to the terminals of a 12 V battery. If another $4\ \Omega$ resistor is connected in parallel with the first, what is the total current drawn from the battery?

5. As more resistors are connected in parallel with a voltage supply, the total:
- current decreases
 - resistance increases
 - power decreases
 - current increases
6. Which of the following formulas is correct for a parallel circuit?
- $E_T = E_1 = E_2 = E_3$
 - $R_T = R_1 = R_2 = R_3$
 - $I_T = I_1 = I_2 = I_3$
 - $P_T = P_1 = P_2 = P_3$
7. Applying Kirchhoff's current law to the circuit in Figure 2, determine the values of currents a, b and c:

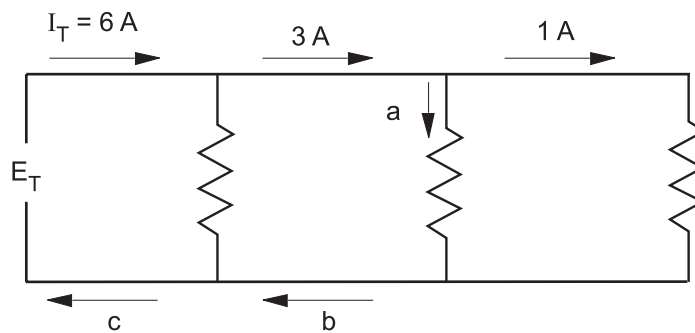


Figure 2—Circuit for Question 7

8. If an open occurs in one branch of a parallel circuit, the line current will:
- increase and the total circuit resistance will decrease
 - decrease and the total circuit resistance will increase
 - decrease and the total circuit resistance will decrease
 - be zero and the total circuit resistance will be infinity

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 5:

Solve problems involving parallel circuits

When analyzing any circuit, it is best to study the schematic diagram before starting calculations. If the problem is presented in words only, and no diagram is given, then draw and carefully label a diagram from the information given.

This initial analysis should point out obvious starting points for solutions. Such starting points occur when two of three required values are given in the diagram. A good first step toward solving parallel circuit problems is to find one of the E values, since E is the same throughout.

Be systematic in your problem solving. A chart is most useful, as it gives you a place to record results and reminds you of what has yet to be solved.

You may also find it easier to mark on the schematic when you have solved for a value.

Example 1: For the circuit in Figure 1, solve for the missing values in Table 1.

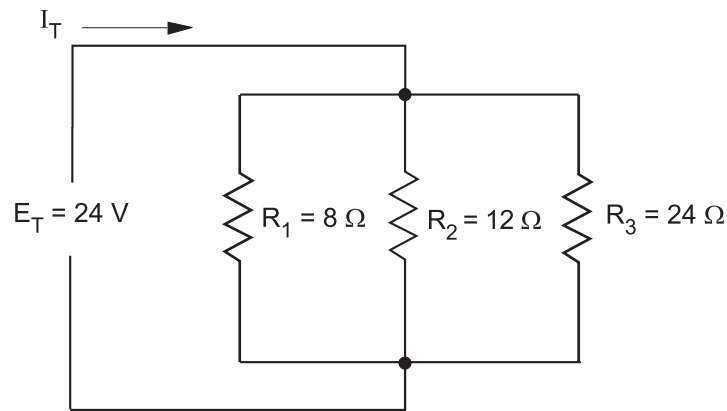


Figure 1—Parallel circuit for Example 1

Table 1 Circuit value chart

	E	I	R	P
Branch 1	_____	_____	$8\ \Omega$	_____
Branch 2	_____	_____	$12\ \Omega$	_____
Branch 3	_____	_____	$24\ \Omega$	_____
Total	24 V	_____	_____	_____

Solution:

Voltage: For a parallel circuit, the total voltage is the same as the voltage across each branch. Therefore:

$$E_T = E_1 = E_2 = E_3 = 24 \text{ V}$$

Current: Use Ohm's law to solve for the currents in each branch:

$$I_1 = \frac{E_1}{R_1} = \frac{24 \text{ V}}{8 \Omega} = 3 \text{ A}$$

$$I_2 = \frac{E_2}{R_2} = \frac{24 \text{ V}}{12 \Omega} = 2 \text{ A}$$

$$I_3 = \frac{E_3}{R_3} = \frac{24 \text{ V}}{24 \Omega} = 1 \text{ A}$$

The total current in a parallel circuit is the sum of the branch currents. Therefore:

$$I_T = I_1 + I_2 + I_3 = 3 \text{ A} + 2 \text{ A} + 1 \text{ A} = 6 \text{ A}$$

Power: Any of the power equations can be used to solve for the power. In this case, use the simplest:

$$P_1 = E_1 \times I_1 = 24 \text{ V} \times 3 \text{ A} = 72 \text{ W}$$

$$P_2 = E_2 \times I_2 = 24 \text{ V} \times 2 \text{ A} = 48 \text{ W}$$

$$P_3 = E_3 \times I_3 = 24 \text{ V} \times 1 \text{ A} = 24 \text{ W}$$

$$P_T = P_1 + P_2 + P_3 = 72 \text{ W} + 48 \text{ W} + 24 \text{ W} = 144 \text{ W}$$

Your chart should now be complete except for the total resistance value.

Resistance: Several different methods can be used to calculate the total resistance in a parallel circuit.

Method 1: Using Ohm's law

If the total circuit voltage and total circuit current are known, then the total circuit resistance can be found by using Ohm's law.

For the circuit in Figure 1:

$$R_T = \frac{E_T}{I_T} = \frac{24 \text{ V}}{6 \text{ A}} = 4 \Omega$$

Note: Total circuit resistance in a parallel circuit is always *less* than the smallest resistance in that circuit. Use this fact to quickly check your calculations.

Method 2: Using the reciprocal equation

You were earlier introduced to this formula, which is used when you know only the resistance values in a circuit:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

By substitution:

$$\frac{1}{R_T} = \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$$

$$\frac{1}{R_T} = \frac{3}{24} + \frac{2}{24} + \frac{1}{24} = \frac{6}{24}$$

$$R = 4 \, \Omega$$

Method 3: Using the product-sum

This method is most commonly used for solving the total resistance of two resistors in parallel. It is really an adaptation of Method 2 and uses the following equation:

$$R_T = \frac{R_1}{R_1} = \frac{R_1}{R_2}$$

To use this method for the circuit in Figure 1, you must go through two steps. Consider first the net resistance of, say, resistors R_2 and R_3 :

$$R_T = \frac{12 \times 24}{12 + 24} = \frac{288}{36} = 8 \, \Omega$$

This value of $8 \, \Omega$ represents the single replacement resistor for that of $12 \, \Omega$ and $24 \, \Omega$ in parallel. You can now consider this value of $8 \, \Omega$ in parallel with R_1 (which is also $8 \, \Omega$).

$$R_T = \frac{8 \times 8}{8 + 8} = \frac{64}{16} = 4 \, \Omega$$

This value of $4 \, \Omega$ is the total resistance of this circuit.

In this example, any of these three methods could have been used. Method 1 would have probably been the easiest to use, since you were asked to solve for all of the currents anyway.

Example 2:**Method 4: Using equal resistances**

In Figure 2, the resistances of each path are known and are equal. The total resistance of the circuit can be found in this case by dividing the value of one resistor by the total number of resistors in parallel.

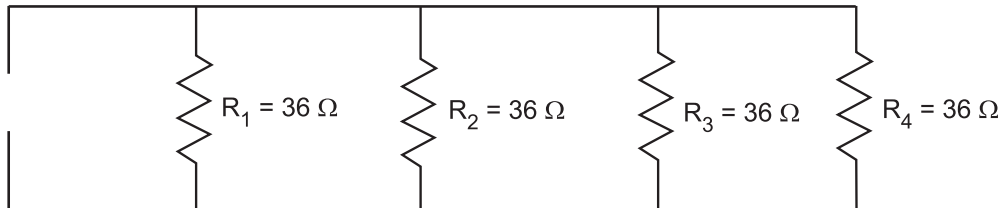


Figure 2—Equal parallel resistors

Solution:

$$R_T = \frac{\text{Value of resistor}}{\text{Number of resistors}}$$

$$= \frac{36\ \Omega}{4} = 9\ \Omega$$

Proportionality and branch currents

Figure 3 illustrates a simple parallel circuit with two branches.

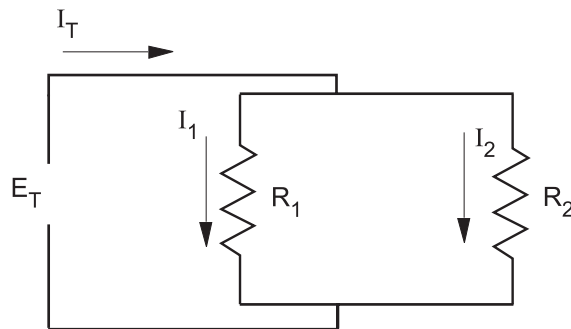


Figure 3—Parallel resistors in a two-branch circuit

Use Ohm's law to solve for voltage across the two resistors in this parallel circuit:

$$E_1 = I_1 \times R_1 \quad \text{and} \quad E_2 = I_2 \times R_2$$

Since $E_1 = E_2$, it follows that:

$$I_1 \times R_1 = I_2 \times R_2$$

The preceding relationships can be manipulated into the following equation:

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

This equation states that the branch currents in a parallel circuit are inversely proportional to their respective resistances. This concept can be used to quickly solve for currents or resistances in parallel circuit problems.

Example 3: A current of 10 A enters a branch of two resistors in parallel. If $R_1 = 20 \Omega$ and the current through $R_1 = 6 \text{ A}$, what is the resistance value of R_2 ?

Solution:

First draw a schematic and label the given data, as shown in Figure 4.

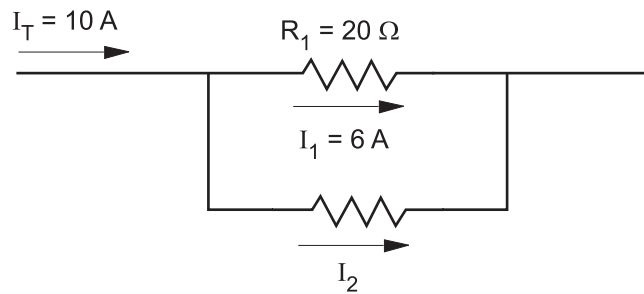


Figure 4—Schematic for Example 3

You can find I_2 by applying Kirchhoff's current law.

$$I_T = I_1 + I_2$$

By substitution:

$$10 \text{ A} = 6 \text{ A} + I_2$$

and thereby:

$$I_2 = 4 \text{ A}$$

Use the inverse proportionality of parallel resistance to solve for R_2 :

$$\frac{R_1}{R_2} = \frac{I_2}{I_1}$$

$$\frac{20 \Omega}{R_2} = \frac{4 \text{ A}}{6 \text{ A}}$$

$$R_2 = \frac{6 \times 20}{4} \Omega = 30 \Omega$$

Example 4: For the lighting branch circuit shown in Figure 5, solve for the values of R_2 and P_2 .

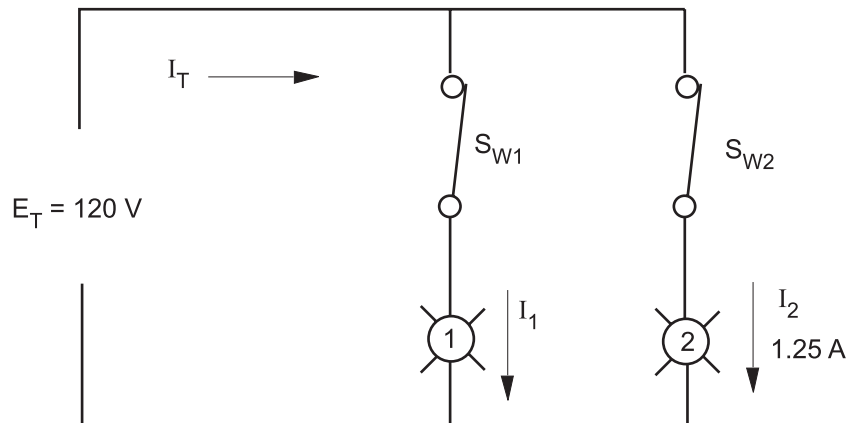


Figure 5—Schematic for Example 4

This is a parallel circuit, so $E_2 = E_T$.

Solution:

Use the branch E_2 , I_2 and Ohm's law to find R_2 :

$$\begin{aligned} R_2 &= \frac{E_2}{I_2} \\ &= \frac{120 \text{ V}}{1.25 \text{ A}} = 96 \, \Omega \end{aligned}$$

That is, the lamp at R_2 has a resistance of $96 \, \Omega$.

To find the power dissipated by the lamp, use either of the power equations; then check with the other:

$$\begin{aligned} P_2 &= E_2 \times I_2 \\ &= 120 \text{ V} \times 1.25 \text{ A} \\ &= 150 \text{ W} \end{aligned}$$

or:

$$\begin{aligned} P_2 &= (I_2)^2 \times R_2 \\ &= (1.25 \text{ A})^2 \times 96 \, \Omega \\ &= 150 \text{ W} \end{aligned}$$

The power used by the lamp is 150 W.

Example 5: The circuit in Figure 6 has a 120 V source voltage and a current of 11 A. It contains two unknown resistances, R_1 and R_2 , and the branch circuit current I_2 is 6 A. Find I_1 , R_T , P_1 , P_2 and P_T .

To answer this question, you must calculate several unknowns. To be systematic, use a table as in Example 1. You may find it helpful to circle the positions of the values you are asked to find.

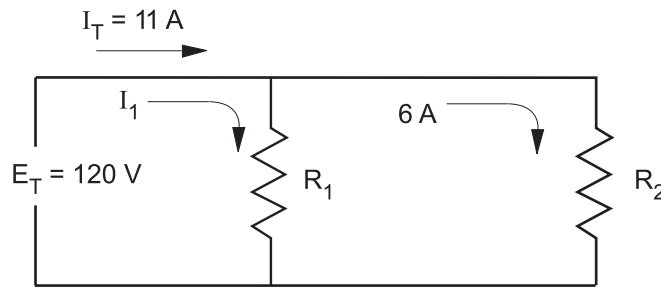


Figure 6—Circuit for Example 5

Solution:

The first step is to transfer information from the schematic to Table 2:

Table 2 Circuit value chart to start Example 5

	E	I	R	P
Branch 1	120 V	_____	_____	_____
Branch 2	120 V	6 A	_____	_____
Total	120 V	11 A	_____	_____

Because you have E_T and I_T , it is easy to solve for R_T :

$$\begin{aligned}
 R_T &= \frac{E_T}{I_T} \\
 &= \frac{120 \text{ V}}{11 \text{ A}} = 10.9 \, \Omega
 \end{aligned}$$

Because you have I_T and I_2 , use Kirchhoff's current law to solve for I_1 :

$$\begin{aligned}
 I_1 &= I_T - I_2 \\
 &= 11 \text{ A} - 6 \text{ A} = 5 \text{ A}
 \end{aligned}$$

Put these values into your table as shown in Table 3:

Table 3 Circuit value chart during calculations for Example 5

	E	I	R	P
Branch 1	120 V	(5 A)	_____	_____
Branch 2	120 V	6 A	_____	_____
Total	120 V	11 A	(10.9 Ω)	_____

The three quantities still to be found are all power values. Since you have both the current and voltage values for each of them, use the general power equation, $P = E \times I$:

$$\begin{aligned}
 P_1 &= E_1 \times I_1 \\
 &= 120 \text{ V} \times 5 \text{ A} = 600 \text{ W}
 \end{aligned}$$

$$\begin{aligned}
 P_2 &= E_2 \times I_2 \\
 &= 120 \text{ V} \times 6 \text{ A} = 720 \text{ W}
 \end{aligned}$$

$$\begin{aligned}
 P_T &= E_T \times I_T \\
 &= 120 \text{ V} \times 11 \text{ A} = 1320 \text{ W}
 \end{aligned}$$

Put these power values into your table as shown in Table 4:

Table 4 Calculated circuit value chart for Example 5

	E	I	R	P
Branch 1	120 V	(5 A)	_____	(600 W)
Branch 2	120 V	6 A	_____	(720 W)
Total	120 V	11 A	(10.9 Ω)	(1320 W)

If you wish, you can finish the chart by finding R_1 and R_2 , both of which must be greater than 10.9 Ω.

$$\begin{aligned}
 R_1 &= \frac{E_1}{I_1} \\
 &= \frac{120 \text{ V}}{5 \text{ A}} = 24 \text{ } \Omega
 \end{aligned}$$

$$\begin{aligned}
 R_2 &= \frac{E_2}{I_2} \\
 &= \frac{120 \text{ V}}{6 \text{ A}} = 20 \text{ } \Omega
 \end{aligned}$$

Table 5 Completed circuit value chart for Example 5

	E	I	R	P
Branch 1	120 V	(5 A)	(24 Ω)	(600 W)
Branch 2	120 V	6 A	(20 Ω)	(720 W)
Total	120 V	11 A	(10.9 Ω)	(1320 W)

Example 6: Calculate the following values for the circuit in Figure 7:

$E_1, E_2, E_3, E_T, I_1,$
 R_1, R_3, R_T and P_T

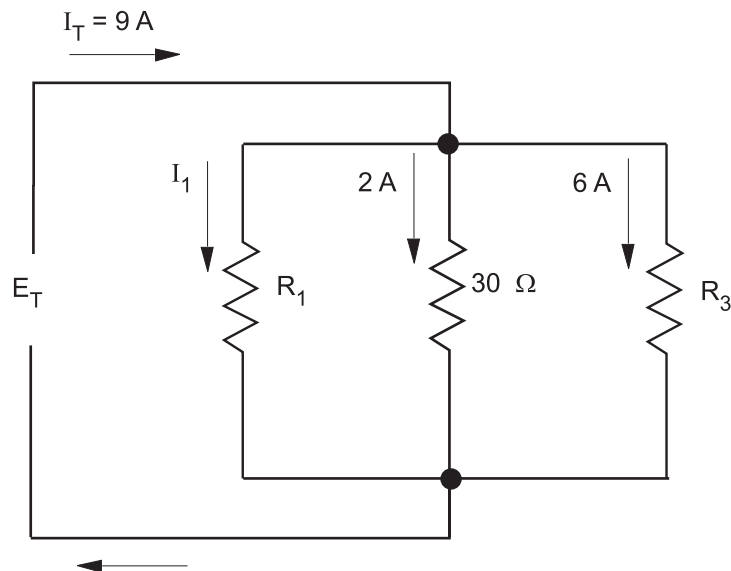


Figure 7—Circuit for Example 6

As before, set up a chart as in Table 6, filling in the given values from the schematic and solving the remaining values.

Table 6 Circuit value chart to start Example 6

	E	I	R	P
Branch 1				
Branch 2		2 A	30 Ω	
Branch 3		6 A		
Total		9 A		

Look for relationships between the known values in Table 6 that can quickly lead to solutions. You see that I_1 and E_2 can be calculated by using Kirchhoff's and Ohm's laws, respectively:

$$\begin{aligned} I_1 &= I_T - I_2 - I_3 \\ &= 9 \text{ A} - 2 \text{ A} - 6 \text{ A} \\ &= 1 \text{ A} \end{aligned}$$

$$\begin{aligned} E_2 &= I_2 \times R_2 \\ &= 2 \text{ A} \times 30 \Omega \\ &= 60 \text{ V} \end{aligned}$$

From this value of E_2 , because this is a parallel circuit:

$$E_1 = E_2 = E_3 = E_T = 60 \text{ V}$$

Insert these values into your chart as in Table 7.

Table 7 Circuit value chart during calculations for Example 6

	E	I	R	P
Branch 1	(60 V)	(1 A)	_____	
Branch 2	(60 V)	2 A	30 Ω	
Branch 3	(60 V)	6 A	_____	
Total	(60 V)	9 A	_____	_____

Now, use Ohm's law to calculate the three required resistances:

$$\begin{aligned} R_1 &= \frac{E_1}{I_1} \\ &= \frac{60 \text{ V}}{1 \text{ A}} = 60 \Omega \end{aligned}$$

$$\begin{aligned} R_3 &= \frac{E_3}{I_3} \\ &= \frac{60 \text{ V}}{6 \text{ A}} = 10 \Omega \end{aligned}$$

$$\begin{aligned} R_T &= \frac{E_T}{I_T} \\ &= \frac{60 \text{ V}}{9 \text{ A}} = 6.67 \Omega \end{aligned}$$

To calculate the total power P_T , you have a choice of methods because you now have values for I_T , E_T and R_T . Use one relationship to calculate P_T and use the other to check your calculation.

$$\begin{aligned} P_T &= E_T \times I_T \\ &= 60 \text{ V} \times 9 \text{ A} \\ &= 540 \text{ W} \end{aligned}$$

$$\begin{aligned} P_T &= (I_T)^2 \times R_T \\ &= 9 \text{ A} \times 9 \text{ A} \times 6.67 \Omega \\ &= 540.27 \text{ W} \\ &\text{which rounds off to } 540 \text{ W} \end{aligned}$$

Insert the calculated values into your chart as shown in Table 8.

Table 8 Circuit value chart with required values for Example 6

	E	I	R	P
Branch 1	(60 V)	(1 A)	(60 Ω)	_____
Branch 2	(60 V)	2 A	30 Ω	_____
Branch 3	(60 V)	6 A	(10 Ω)	_____
Total	(60 V)	9 A	(6.67 Ω)	(540 W)

Sometimes it is a good idea to check a few calculations by alternative methods. See if you can determine some of the values in Example 5 using different calculation methods.

Example 7: In Figure 8, a current of 5 A is branched in such a way that only 1 mA flows in the branch containing R_1 . Find R_2 .

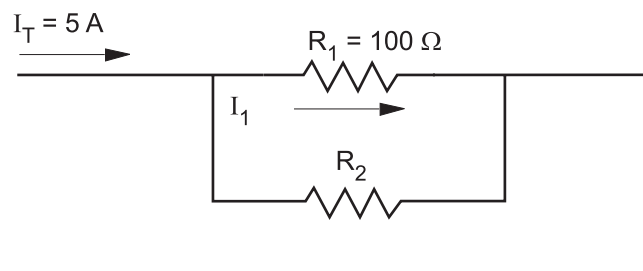


Figure 8—Circuit for Example 7

Solution:

First, list all the known values:

$$I_T = 5 \text{ A}$$

$$R_1 = 100 \, \Omega$$

$$I_1 = 1 \text{ mA}$$

$$= 0.001 \text{ A}$$

Use Kirchhoff's law to find I_2 :

$$I_2 = I_T - I_1$$

$$= 5 \text{ A} - 0.001 \text{ A}$$

$$= 4.999 \text{ A}$$

Now apply the rule of inverse proportionality of parallel resistances:

$$\frac{R_1}{R_2} = \frac{I_2}{I_1}$$

$$\frac{100 \, \Omega}{R_2} = \frac{4.999 \text{ A}}{0.001 \text{ A}}$$

$$R_2 = \frac{100 \, \Omega \times 0.001 \text{ A}}{4.999}$$

$$= 0.02 \, \Omega$$

The parallel resistance is $0.02 \, \Omega$.

This example shows a procedure frequently used to find the size of shunt needed to extend the range of an ammeter. Ammeter sensitivity and determination of shunt size are covered more fully in Learning Guide E1: Use Analog Meters.

Example 8: An ammeter has a sensitivity of 1 mA and an internal resistance of $100 \, \Omega$. Determine the amount of shunt resistance needed to allow the meter to read 10 A (Figure 9).

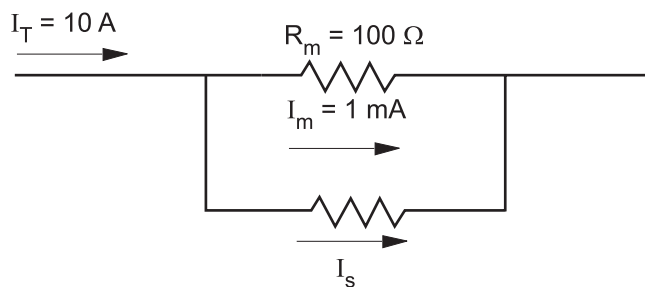


Figure 9—Ammeter shunt circuit for Example 8

Solution:

Let R_m be the meter resistance and R_s the shunt resistance.

First, list all known values:

$$I_T = 10 \text{ A}$$

$$I_m = 1 \text{ mA}$$

$$= 0.001 \text{ A}$$

$$R_m = 100 \Omega$$

Use Kirchhoff's current law to calculate I_s :

$$I_T = I_m + I_s$$

$$I_s = 10 \text{ A} - 0.001 \text{ A}$$

$$= 9.999 \text{ A}$$

Use the proportionality rule to find shunt size R_s :

$$\frac{R_m}{R_s} = \frac{I_s}{I_m}$$

$$\frac{100 \Omega}{R_s} = \frac{9.999 \text{ A}}{0.001 \text{ A}}$$

$$R_s = 0.01 \Omega$$



Now do Self-Test 5 and check your answers.

Self-Test 5

1. What is the total resistance of a circuit that contains three resistors in parallel, if $R_1 = 20\ \Omega$, $R_2 = 30\ \Omega$ and $R_3 = 60\ \Omega$?
-

2. Calculate each of the following values for the circuit in Figure 1:

- a. E_1 _____
 b. E_2 _____
 c. E_T _____
 d. I_2 _____
 e. R_T _____
 f. P_T _____

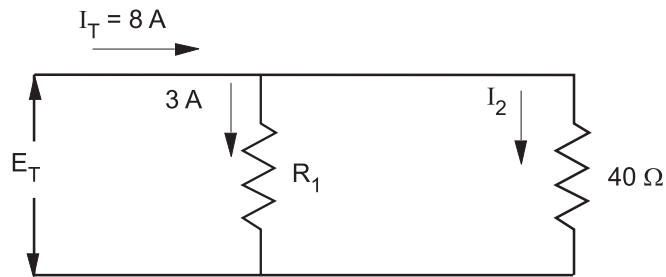


Figure 1—Circuit for Question 2

3. An ammeter's sensitivity is 1 mA and its internal resistance is $50\ \Omega$. What size shunt resistor is needed to extend the range of the meter to 20 A?
-

4. Calculate each of the following values for the circuit in Figure 2:

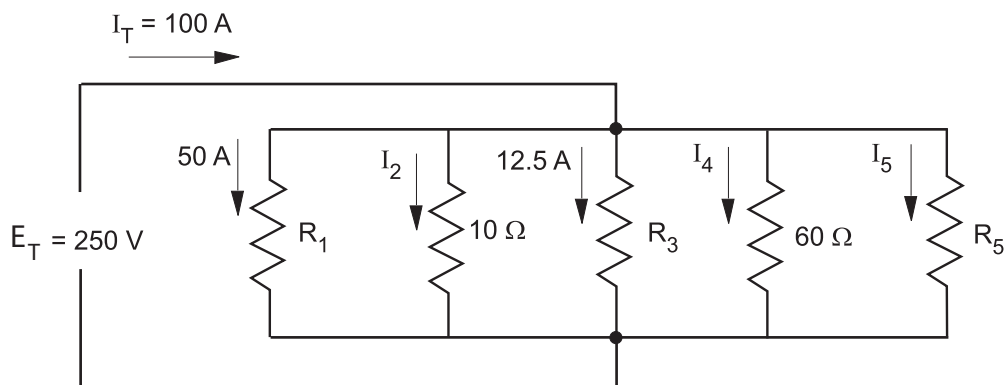


Figure 2—Circuit for Question 4

- a. E_1 _____
- b. R_1 _____
- c. I_2 _____
- d. R_3 _____
- e. I_4 _____
- f. R_5 _____
- g. R_T _____
- h. P_5 _____
- i. P_T _____

5. The total resistance of the circuit in Figure 3 is:

- a. $90\ \Omega$
- b. $30\ \Omega$
- c. $20\ \Omega$
- d. $3\ \Omega$

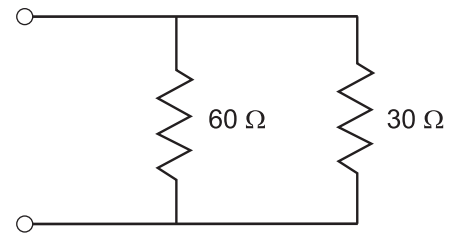


Figure 3—Circuit for Question 5

6. Referring to the circuit in Figure 4, complete Table 1:

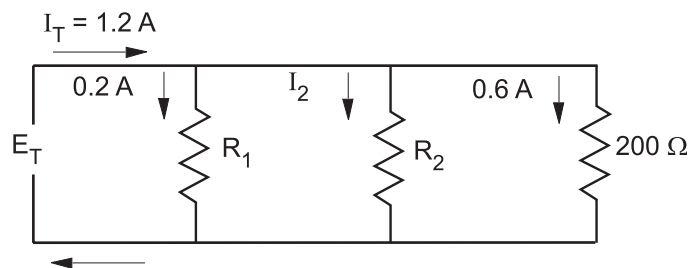


Figure 4—Circuit for Question 6

Table 1 Circuit value chart for Question 6

	E	I	R	P
Branch 1				
Branch 2				
Branch 3				
Total				

7. Calculate each of the following values for the circuit in Figure 5:

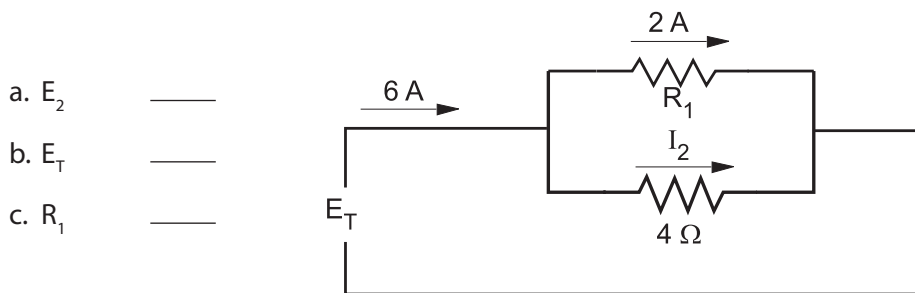


Figure 5—Circuit for Question 7

8. The total resistance of two parallel-connected resistors is $6\ \Omega$. If one of the resistors has a resistance of $12\ \Omega$, what is the resistance of the other?

9. In the circuit in Figure 6, $R_1 = R_3$ and the ammeter reads 8 A. Determine the power developed by each resistor and the total circuit power.

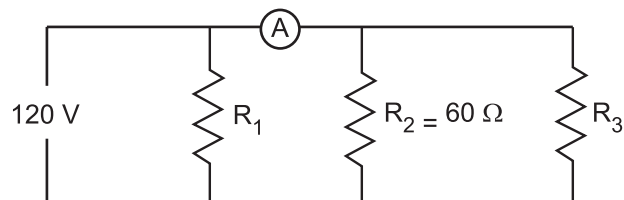


Figure 6—Circuit for Question 9

10. When two equal resistors are connected in parallel to a 120-volt supply, the resulting line current is 10 amps. If these same two resistors were connected in series to the same supply, what would the total line current be?
-

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 6:

Describe the effects of connecting voltage sources in parallel

Voltage sources are connected in parallel whenever it is necessary to deliver a current output greater than the current output a single source of supply can provide, without increasing voltage across a load.

- Power sources are connected *in series* to increase the *voltage output*.
- Conversely, power sources are connected *in parallel* to increase the *current capacity*.

An advantage of parallel-connected power sources is that one source can be removed for maintenance or repairs while reduced power to the load is maintained. Other factors affect power sources operated in parallel and these will be explained to you in future studies.

For parallel batteries, current capacity is equal to that of one *battery* multiplied by the number of parallel *batteries*. If a 6 V battery has a maximum current output of 1 A, and if it is necessary to supply a load requiring 2 A, then you can connect a second 6 V battery in parallel with the first as in Figure 1.

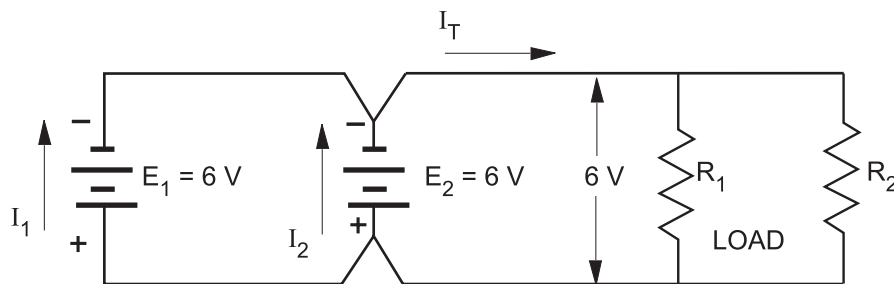


Figure 1—Two batteries in parallel

Whenever batteries or other power sources such as transformers or generators are to be connected in parallel, it is *very important* that the power sources have the following:

- *the same terminal voltages*

A lower voltage source connected to a higher one acts as a load itself, rather than helping share the real load current with the other sources of supply.

- *properly connected polarity*

Like terminals of the power sources must be connected together: that is, positive-to-positive and negative-to-negative.

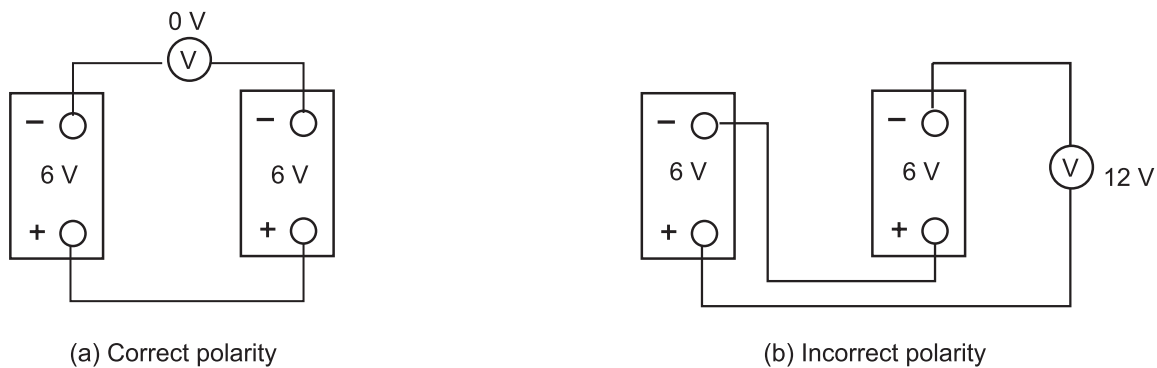


Figure 2—Polarity test

Before connecting two voltage sources in parallel, do a simple voltmeter test for correct polarity.

1. Tie one side of the power sources together.
2. Before connecting the paralleling jumper between the remaining two terminals, insert a voltmeter between these two points. See Figure 2.
3. If the polarity is incorrect (Figure 2b), the voltmeter indicates two times the source voltage because the equal emfs aid each other. Do NOT connect across these terminals.



CAUTION! Since there is a difference in potential between these two points, connecting a paralleling jumper between them would result in a SHORT CIRCUIT!

4. If the polarity is correct (Figure 2a), then the voltmeter indicates 0 V because the emfs oppose each other. You may connect a paralleling jumper between these two points.

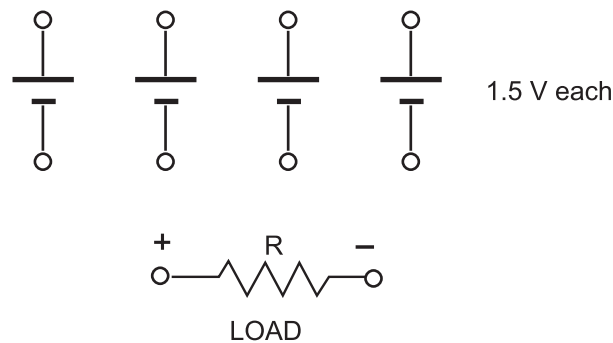


Now do Self-Test 6 and check your answers.

Self-Test 6

1. Before connecting two 12 V batteries in parallel, you perform a voltmeter test to confirm proper polarity. What does the voltmeter read if the batteries are connected with
 - a. incorrect polarity? _____
 - b. correct polarity? _____
2. What is the main advantage of parallel-connected power sources over series-connected power sources?

3. Four 1.5 V cells are connected in a bank so that they will deliver an output of 3 V. Complete the diagram in Figure 3 to show how they should be connected.

**Figure 3—Components for Question 3**

4. If each of the cells in Question 3 is rated at 2 A capacity, what is the total available current from the bank?

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 7:

Describe the characteristics of combination circuits

In practical electrical applications, pure series circuits and pure parallel circuits are seldom encountered. Most practical electrical circuits are *combination circuits* that combine both series- and parallel-connected elements. Some texts refer to combination circuits as *series-parallel circuits*.

Although some combination circuits are complex, they can be systematically broken down into parallel and series components. The laws that apply to series circuits and parallel circuits separately can be applied to the appropriate parts of a combination circuit.

There are endless possibilities for constructing a combination circuit. Since every combination circuit contains both series- and parallel-connected devices, you must be able to identify the series and parallel components within a combination circuit.

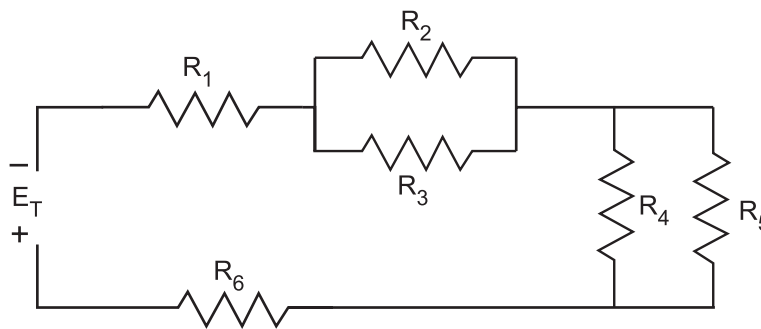


Figure 1—Combination circuit

In the combination circuit in Figure 1, you can see that resistors R_2 and R_3 are in parallel, but only with each other. Resistors R_4 and R_5 are also in parallel, but only with each other. To complete the circuit loop, both of these two parallel portions are in series with each other, as well as with resistors R_1 and R_6 .

Quite often, when solving problems, it is necessary to break a combination circuit down into a *series equivalent circuit*.

To do this, use the product-over-sum method of calculating the equivalent resistance of pairs of parallel resistances in the circuit.

For example, the parallel resistors R_2 and R_3 in Figure 1 can be calculated as an equivalent single resistor, shown as R_{23} in Figure 2. Also, the other parallel portion of the circuit in Figure 1 consisting of R_4 and R_5 can be shown as the single equivalent resistor R_{45} in Figure 2.

After replacing the two pairs of parallel resistances by their equivalent resistances, the original combination circuit in Figure 1 looks like the simple series circuit in Figure 2.

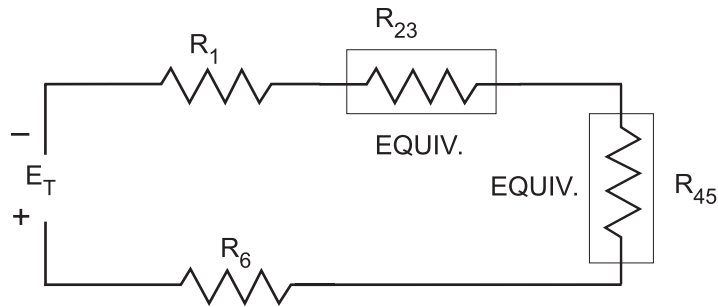


Figure 2—Redrawn equivalent series circuit for Figure 1

The laws of a series circuit can be applied to this circuit to solve for the total circuit resistance, the total line current and the voltage drops across series components.

Quite often when solving combination-circuit problems, you will attempt to redefine the parallel circuit portions of a circuit so that you end up with a simple series circuit equivalent.

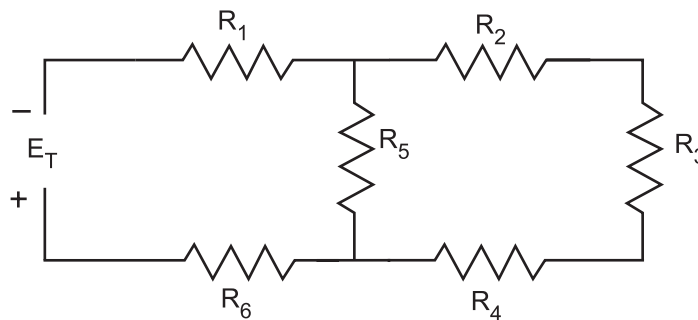


Figure 3—Another combination circuit

To convert the combination circuit in Figure 3 to a simplified series equivalent, use a systematic approach:

First, calculate the equivalent resistances:

1. Note that resistors R_2 , R_3 and R_4 form a small series circuit, which is in parallel with resistor R_5 .
2. Calculate resistors R_2 , R_3 and R_4 as a single equivalent resistor.
3. R_{234} is shown in Figure 4a, connected in parallel with resistor R_5 .
4. Simplify this parallel combination of R_{234} with R_5 into a single equivalent resistor, R_{2345} .
5. Connect R_{2345} in series with the rest of the circuit as shown in Figure 4b.

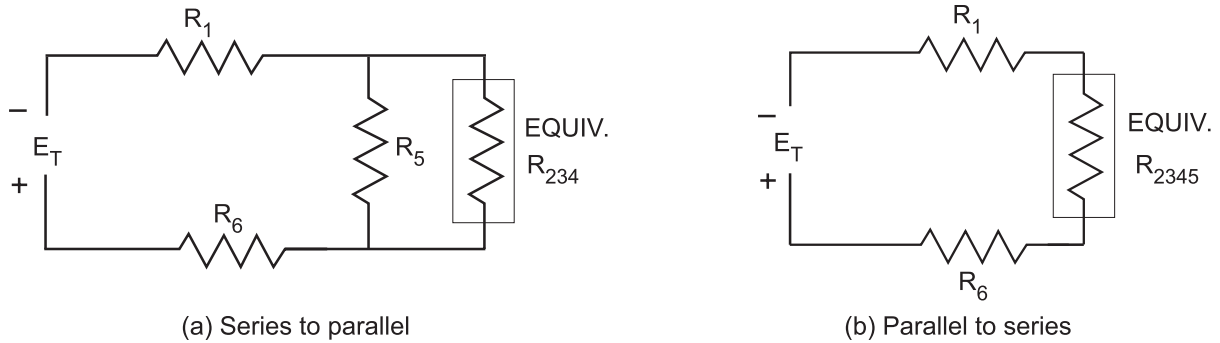


Figure 4—Redrawn equivalent circuits for Figure 3

6. As you can see, you are now left with a simple series circuit consisting of R_1 , R_6 and the reduced loop equivalent.
7. Determine the net resistance R_T of the combination circuit using the series circuit law ($R_T = R_1 + R_{2345} + R_6$).

Now calculate the total current and individual voltages:

8. Once the total circuit resistance is found, solve for the total circuit current (I_T) by applying Ohm's law ($I = E \div R$).
9. Now you may use this current to determine the individual voltage drops in the reduced equivalent circuit. See Figure 5.

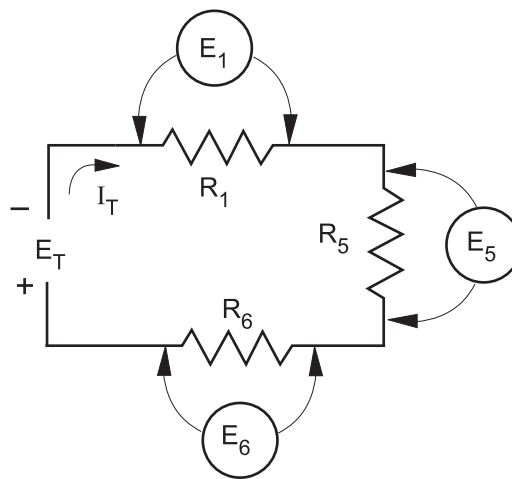


Figure 5—Voltages in the equivalent circuit

10. Calculate the individual voltage drops as:

$$E_1 = I_T \times R_1$$

$$E_5 = I_T \times R_{2345}$$

$$E_6 = I_T \times R_6$$

Of course, the sum of $E_1 + E_5 + E_6$ must equal the value of E_T according to series circuit laws.

Now, calculate the branch currents:

11. Use the voltage drop (E_5) appearing across the equivalent resistance portion of the circuit, to determine the branch currents illustrated in Figure 6.

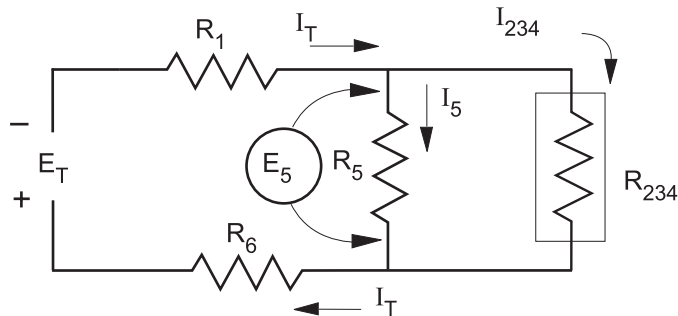


Figure 6—Branch currents

12. For this parallel portion of this combination circuit, the voltage drops across the two branches are the same ($E_5 = E_{234}$).
13. Recall Kirchhoff's current law, $I_T = I_5 + I_{234}$, for the parallel portion.

Once the branch currents are determined, calculate the individual voltage drops across the original series resistors:

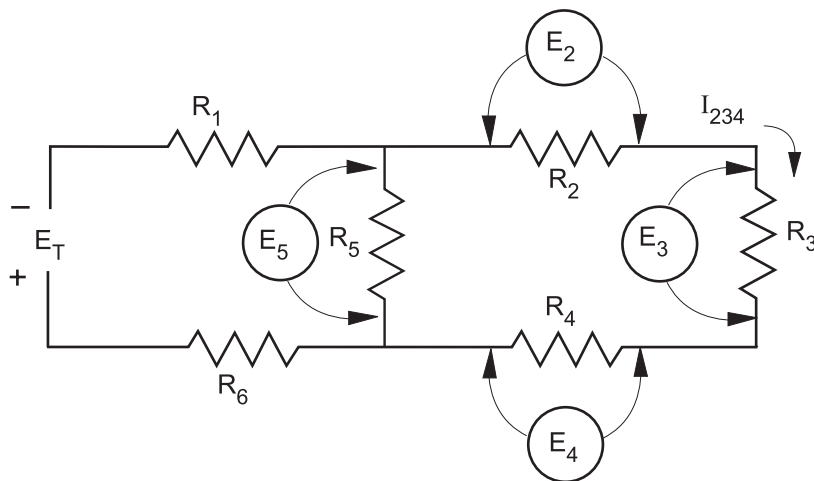


Figure 7—Voltage drops

14. Figure 7 shows the original series branch circuit of Figure 3. Apply Ohm's law to solve for the individual voltage drops:

$$E_2 = I_{234} \times R_2$$

$$E_3 = I_{234} \times R_3$$

$$E_4 = I_{234} \times R_4$$

Also,

$$E_5 = E_2 + E_3 + E_4 \text{ for this portion of the circuit.}$$

Determine component polarities:

15. Use the current paths for this combination circuit to mark the polarity for the individual resistors. Study the schematic shown in Figure 8.

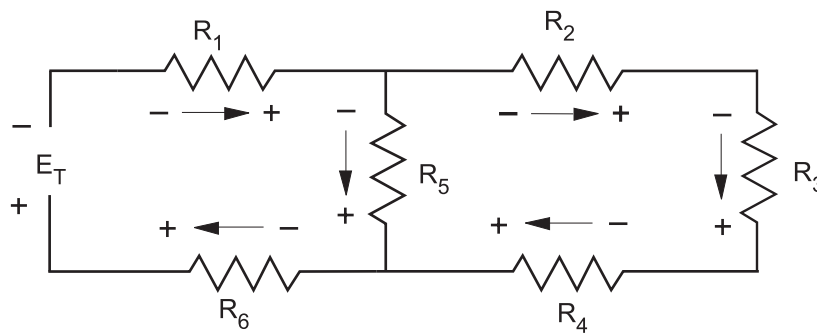


Figure 8—Marking of polarity

Determine power:

Power in a combination circuit is determined no differently than it was in pure series, or in pure parallel circuits. The total power can be solved by using line (total) values. The individual power values can be solved by using individual resistive values, and applying any of the power equations.



Now do Self-Test 7 and check your answers.

Self-Test 7

1. Neglecting any resistance in the connecting wires, the circuit in Figure 1 is best classified as a:
 - a. series circuit
 - b. parallel circuit
 - c. combination circuit

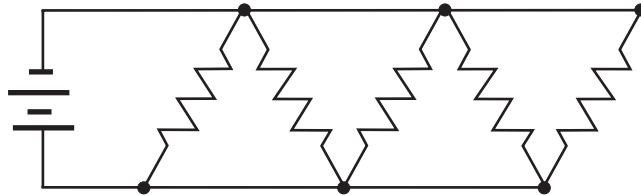


Figure 1—Circuit for Question 1

2. According to the ammeter readings for the combination circuit shown in Figure 2, determine the current flow through resistor:
 - a. R_5
 - b. R_2
 - c. R_3

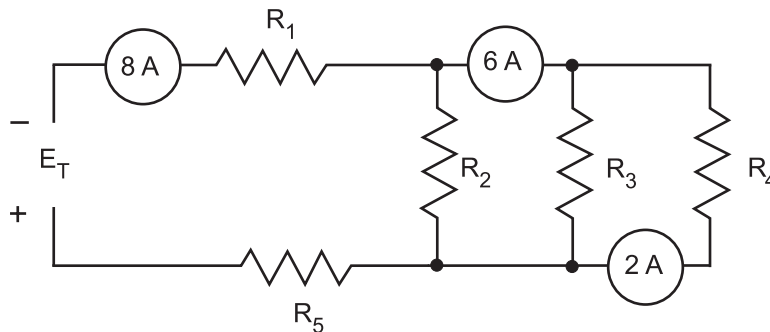


Figure 2—Circuit for Question 2

3. In Figure 3, mark each of the resistors with correct polarity.

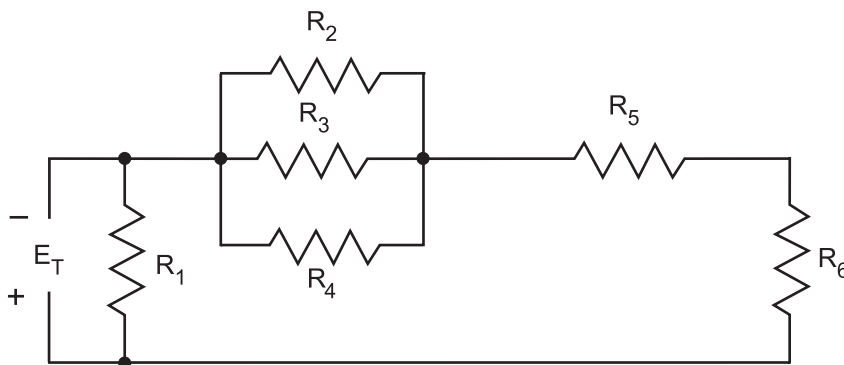


Figure 3—Circuit for Question 3

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 8:

Solve problems involving combination circuits

To solve problems dealing with combination circuits, you must first be able to work effectively with series circuits and parallel circuits. This is important because you must be able to apply the laws of series and parallel circuits to any combination circuit.

Then you need to use a systematic approach to solving the problem. Usually this means redrawing the circuit to equivalent circuits, as this makes it easier to solve for total resistance, current or voltage values. These calculated values can then be reapplied to the original circuit.

Equivalent resistance circuits

All resistive circuits containing two or more resistors can be reduced to a single resistor (R_T) that represents the total resistance. You have done this already with series circuits: you found R_T by adding the individual resistances. In parallel circuits, you may have used several different methods—most often the reciprocal method—to solve for R_T .

As you are learning in Learning Task 7, when you are working with combination circuits, the usual procedure is to find equivalent resistor values for parallel components and to redraw the circuit to show resistive loads in series. It is then a simple matter of adding equivalent resistive values directly to find the net resistance (R_T).

Example 1: Determine the total resistance of the circuit in Figure 1.

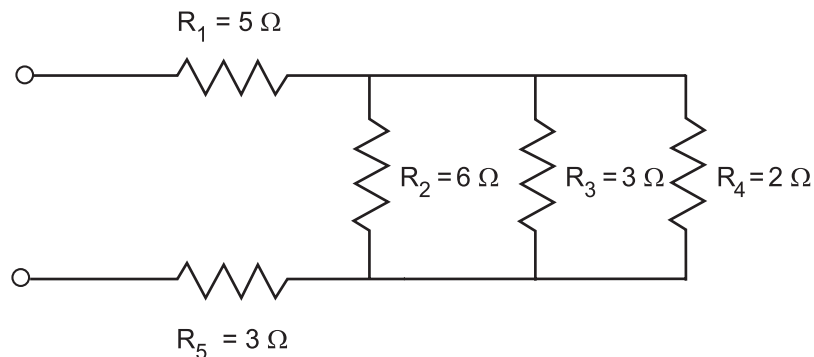


Figure 1—Circuit for Example 1

Solution:

You can see that R_2 , R_3 and R_4 are in parallel.

Find the equivalent resistance for these three resistances and call it R_6 .

Use the reciprocal resistances method.

$$\begin{aligned}
 \frac{1}{R_6} &= \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \\
 &= \frac{1}{6} + \frac{1}{3} + \frac{1}{2} \\
 &= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} \\
 R_6 &= 1 \, \Omega
 \end{aligned}$$

That is, resistances R_2 , R_3 and R_4 can be replaced by a single $1 \, \Omega$ resistance. The formal way to write this down is:

$$R_2 // R_3 // R_4 = R_6 = 1 \, \Omega$$

where the symbol $//$ means “in parallel with.”

Now redraw the circuit as in Figure 2. Show the equivalent resistance in series with the remaining two resistances in the circuit.

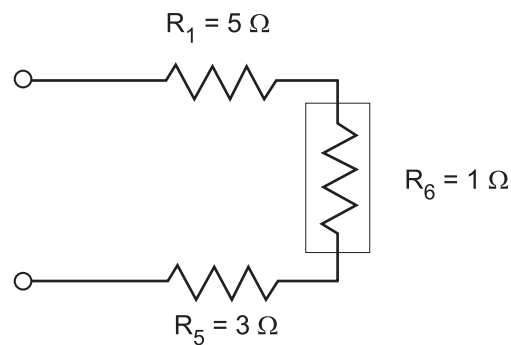


Figure 2—Equivalent circuit for Example 1

Now the circuit can be viewed as a simple series circuit, which allows you to find the total circuit resistance, R_T , by adding R_1 , R_5 and R_6 :

$$\begin{aligned}
 R_T &= R_1 + R_5 + R_6 \\
 &= 5 \, \Omega + 3 \, \Omega + 1 \, \Omega \\
 &= 9 \, \Omega
 \end{aligned}$$

Example 2: Reduce the circuit in Figure 3 to its equivalent resistance, R_T .

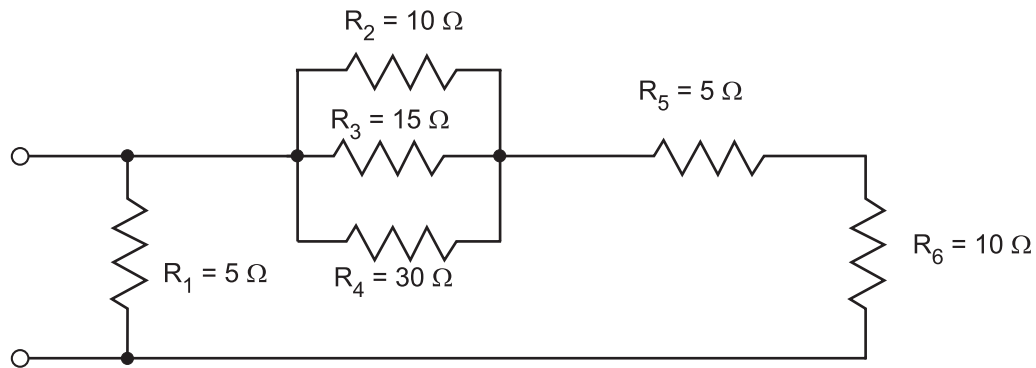


Figure 3—Circuit for Example 2

Solution:

Resistances R_2 , R_3 and R_4 are in parallel. First find the single equivalent resistance R_7 that could replace them.

$$\begin{aligned}\frac{1}{R_7} &= \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \\ &= \frac{1}{10} + \frac{1}{15} + \frac{1}{30} \\ &= \frac{3 + 2 + 1}{30} \\ R_7 &= 5 \Omega\end{aligned}$$

Redraw the circuit as in Figure 4, replacing $R_2 // R_3 // R_4$ with $R_7 = 5 \Omega$.

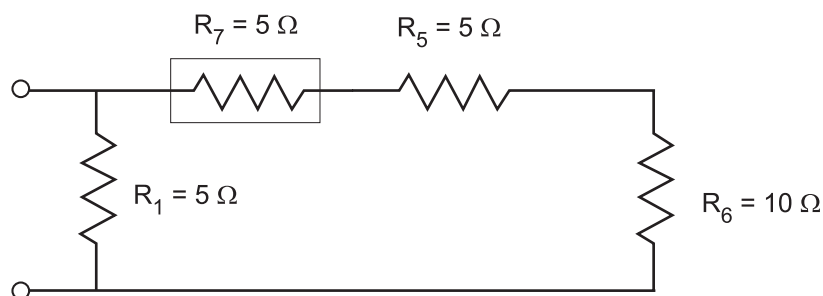


Figure 4—First equivalent circuit for Example 2

R_7 , R_5 and R_6 receive the same electron flow, which means they are in series. They can be replaced by a single resistance R_8 calculated by adding them:

$$\begin{aligned} R_8 &= R_7 + R_5 + R_6 \\ &= 5 \, \Omega + 5 \, \Omega + 10 \, \Omega \\ &= 20 \, \Omega \end{aligned}$$

Redraw the circuit again as in Figure 5, replacing R_7 , R_5 and R_6 with R_8 .

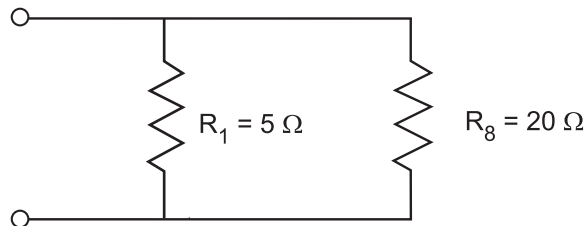


Figure 5—Second equivalent circuit for Example 2

Figure 5 is a simple parallel circuit. Now find the equivalent resistance for R_1 and R_8 , which is R_T . For practice, use the product-sum method:

$$\begin{aligned} R_T &= \frac{R_1 \times R_8}{R_1 + R_8} \\ &= \frac{5 \times 20}{5 + 20} \, \Omega = \frac{100}{25} \, \Omega = 4 \, \Omega \end{aligned}$$

Example 3: Reduce the circuit in Figure 6 to its equivalent resistance value, R_T .

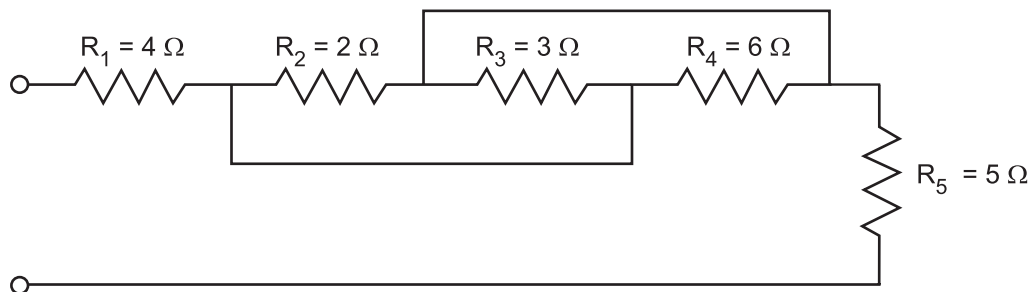


Figure 6—Circuit for Example 3

Give the circuit in Figure 6 a node analysis to discover the common points among the resistances. Label the common nodes a and b, as shown in Figure 7.

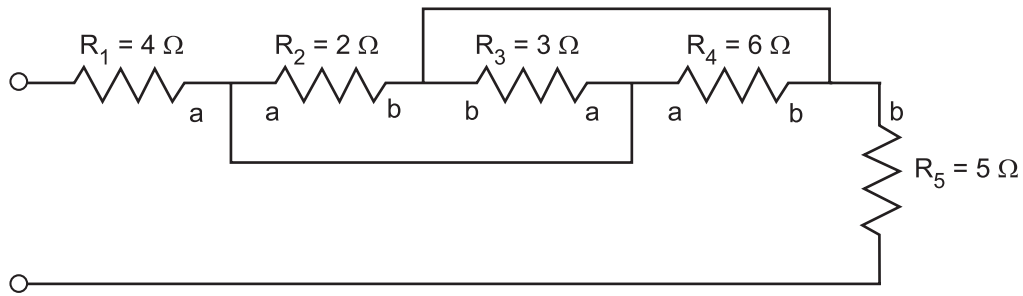


Figure 7—Circuit for Example 3 showing labelled common nodes

Resistors R_2 , R_3 and R_4 all have the same end labels, a and b, so they must be in parallel. Use the reciprocal equation to find resistance R_6 to replace them.

$$\begin{aligned}\frac{1}{R_6} &= \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \\ &= \frac{1}{2} + \frac{1}{3} + \frac{1}{6}\end{aligned}$$

$$R_6 = 1 \Omega$$

Now redraw the circuit as in Figure 8, replacing R_2 , R_3 and R_4 with R_6 .

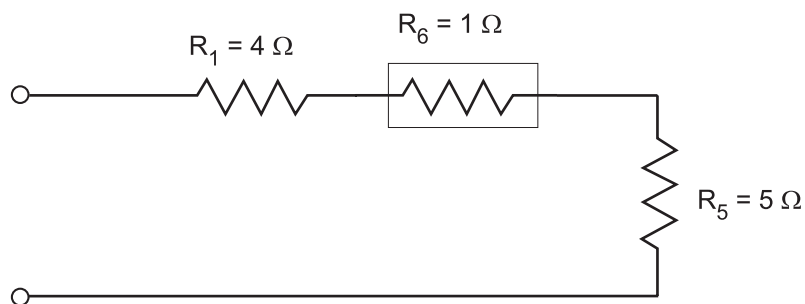


Figure 8—Equivalent circuit for Example 3

Figure 8 is a simple series circuit, so find R_T by adding R_1 , R_5 and R_6 .

$$\begin{aligned}R_T &= R_1 + R_5 + R_6 \\ &= 4 \Omega + 1 \Omega + 5 \Omega \\ &= 10 \Omega\end{aligned}$$

Example 4: Find R_T for the circuit in Figure 9.

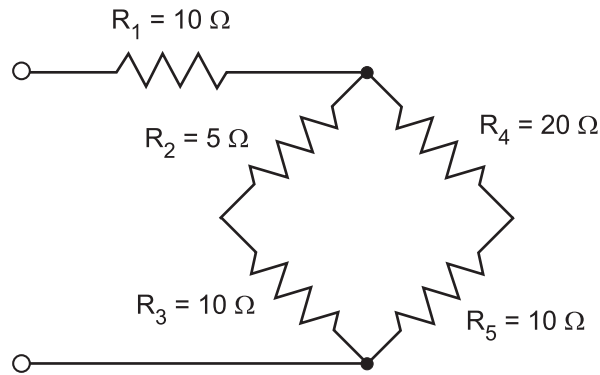


Figure 9—Circuit for Example 4

Solution:

Sometimes, a useful step in solving a combination circuit problem is to redraw the circuit in a different form. Figure 10 is electrically identical to Figure 9.

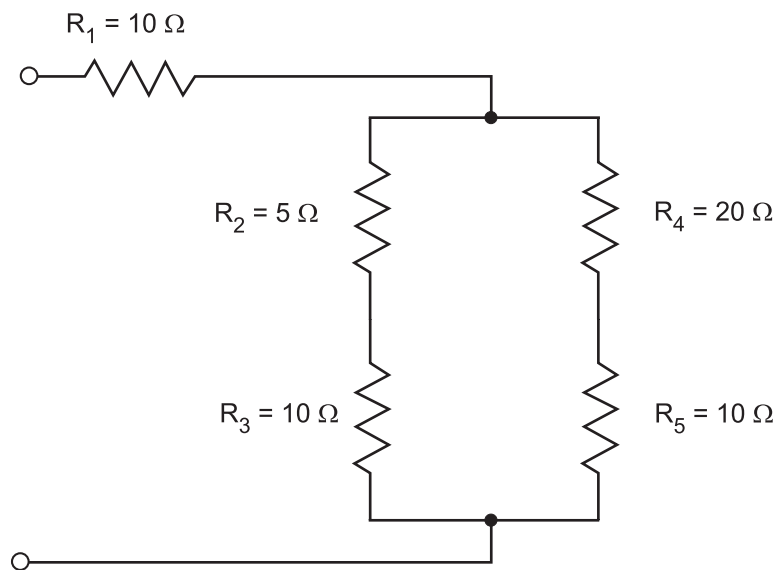


Figure 10—Redrawn circuit for Example 4

The rest of the solution to this problem is shown in diagram form only in Figure 11, parts a, b, c and d. Try to follow the steps, doing your own calculations. Check them with the following summary.

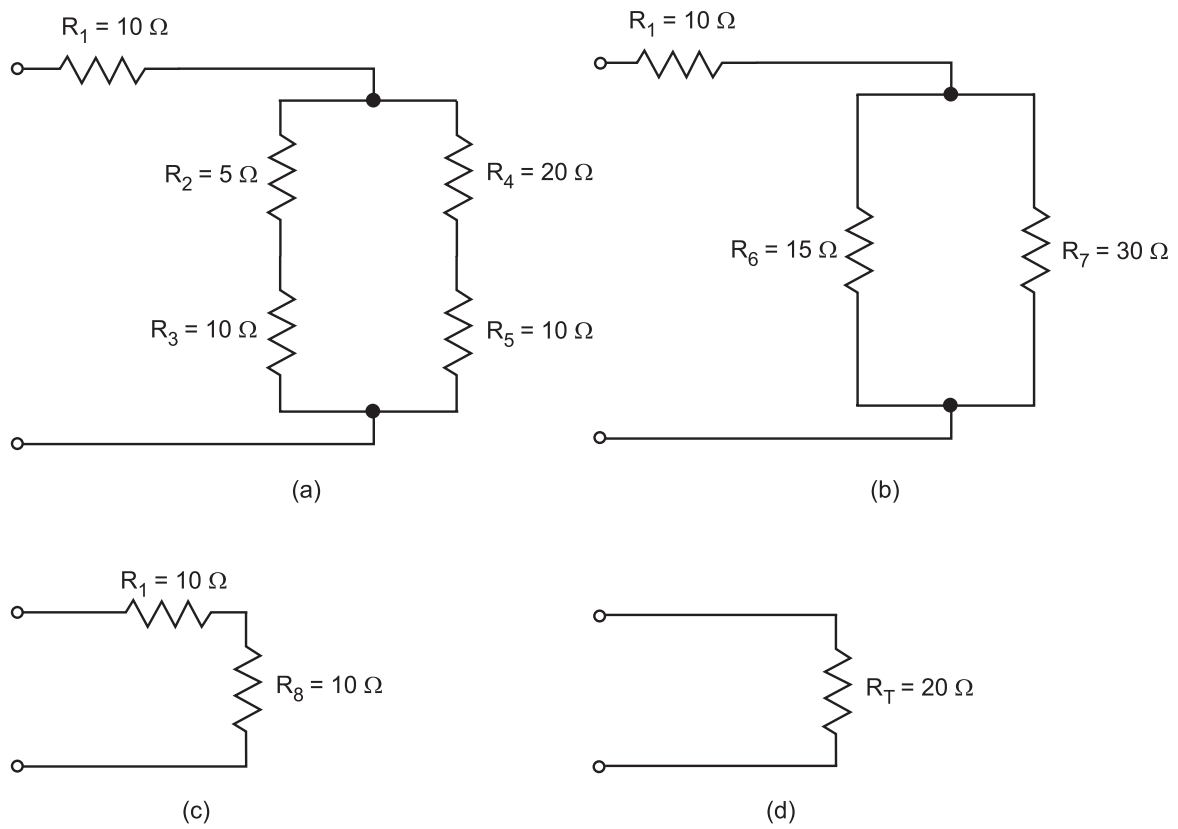


Figure 11—Diagrammatic solution for Example 4

In summary:

1. R_2 and R_3 are in series and are added to produce R_6 .
2. Also, R_4 and R_5 are in series and are added to produce R_7 .
3. These steps combined result in Figure 11b.
4. The two new resistances $R_6 // R_7 = R_8$.
5. This step results in Figure 11c.
6. R_1 and R_8 are in series and are added to produce $R_T = 20\ \Omega$ (Figure 11d).

Solving combination circuits

When solving combination circuits for current, voltage and power, keep in mind the basic laws for series circuits and parallel circuits.

Remember that:

<i>Series circuits</i>	Voltage changes over a resistance (resistive load); current stays the same through a resistive load.
<i>Parallel circuits</i>	Current changes at nodes; voltage remains the same across parallel resistances.
<i>Kirchhoff's voltage law</i>	The sum of the voltage drops in a circuit is equal to the source voltage.
<i>Kirchhoff's current law</i>	The current entering a circuit branch is equal to the current exiting the circuit branch.

The following examples of combination circuit problems will test your knowledge of series and parallel circuits.

Example 5: In the circuit in Figure 12, find each of the following eight values:

R_T , I_T , P_T ;

E_1 , E_2 , E_3 ;

I_2 and I_3 .

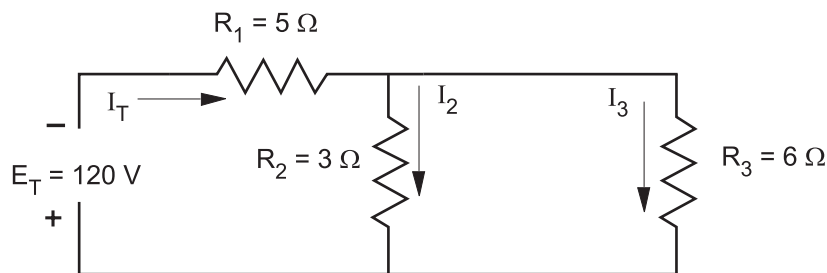


Figure 12—Circuit for Example 5

Use a chart like Table 1 to keep track of information. You may find it helpful to fill in known values and leave blanks where values are to be calculated.

Table 1 Circuit value chart to start Example 5

	E	I	R	P
Resistor 1			5 Ω	
Resistor 2			3 Ω	
Resistor 3			6 Ω	
Total	120 V			

First, find R_T :

$R_2/R_3 = R_{23}$, so use the product and sum method to calculate R_{23} :

$$R_{23} = 2 \Omega$$

R_1 is in series with R_{23} . So, adding:

$$R_T = 5 \Omega + 2 \Omega = 7 \Omega$$

Then, find I_T :

Using Ohm's law:

$$\begin{aligned} I_T &= 120 \text{ V} \div 7 \Omega \\ &= 17.14 \text{ A} \end{aligned}$$

Now find the voltages across the resistors:

Use Ohm's law again to find the voltage drop E_1 across R_1 :

$$E_1 = 17.14 \text{ A} \times 5 \Omega = 85.7 \text{ volts}$$

Because of the voltage drop across R_1 , E_2 and E_3 must be less than E_T .

Use Kirchhoff's voltage law to calculate E_2 :

$$E_2 = 120 \text{ V} - 85.7 \text{ V} = 34.3 \text{ V}$$

R_2 and R_3 are in parallel, so that

$$E_3 = E_2 = 34.3 \text{ V}$$

Now *update your chart* as shown in Table 2. Calculated values are in brackets; given values are unbracketed; blanks mark the values still to be calculated.

Table 2 Circuit value chart during calculations for Example 5

	E	I	R	P
Resistor 1	(85.7 V)		5 Ω	
Resistor 2	(34.3 V)	_____	3 Ω	
Resistor 3	(34.3 V)	_____	6 Ω	
Total	120 V	(17.14 A)	(7 Ω)	_____

You have to fill only three more blanks in the chart.

Find the branch currents:

Use Ohm's law to solve for I_2 :

$$I_2 = 34.3 \text{ V} \div 3 \Omega = 11.43 \text{ A}$$

You can use either Ohm's law or Kirchhoff's current law to find I_3 .

Using Ohm's law:

$$I_3 = 34.3 \text{ V} \div 6 \Omega = 5.716 \text{ A}$$

Using Kirchhoff's current law:

$$I_3 = 17.14 \text{ A} - 11.43 \text{ A} = 5.71 \text{ A}$$

Note that the difference in the two answers is due to rounding off during calculation of I_1 and I_T and is insignificant.

Finally, find the power:

Use one of the power equations to find total power dissipated by the circuit.

Using $P_T = I_T \times E_T$:

$$P_T = 17.14 \text{ A} \times 120 \text{ V} = 2056.8 \text{ W}$$

You can now complete your chart as in Table 3:

Table 3 Completed circuit value chart for Example 5

	E	I	R	P
Resistor 1	(85.7 V)		5 Ω	
Resistor 2	(34.3 V)	(11.43 A)	3 Ω	
Resistor 3	(34.3 V)	(5.71 A)	6 Ω	
Total	120 V	(17.14 A)	(7 Ω)	(2056.8 W)

Example 6: For the circuit in Figure 13, calculate each of the following:

$$R_T, I_T$$

$$E_1, E_2, E_3, E_4, E_5$$

$$I_1, I_2, I_3, I_4 \text{ and } I_5.$$

Record the values in a chart like Table 4.

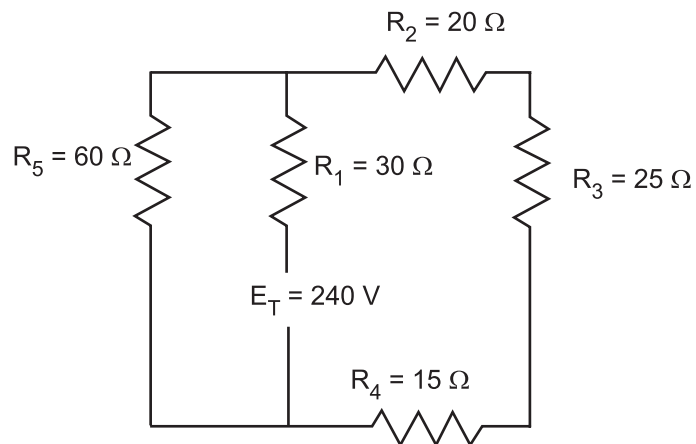


Figure 13—Circuit for Example 6

Table 4 Circuit value chart to start Example 6

	E	I	R
Resistor 1	_____	_____	
Resistor 2	_____	_____	
Resistor 3	_____	_____	
Resistor 4	_____	_____	
Resistor 5	_____	_____	
Total		_____	_____

If the circuit looks confusing, redraw it by moving R_5 to the right-hand side, so that it looks like Figure 14.

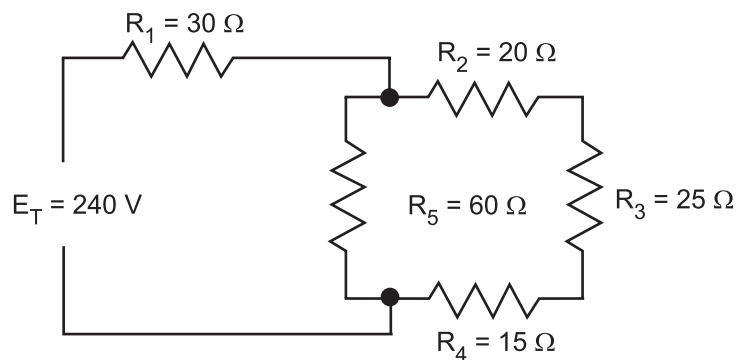


Figure 14—Redrawn circuit for Example 6

First, find R_1 :

R_2 , R_3 and R_4 are in series with one another and can be replaced with resistance R_6 where, by addition:

$$R_6 = 20\ \Omega + 25\ \Omega + 15\ \Omega = 60\ \Omega$$

R_6 is in parallel with R_5 . Use the reciprocal method to calculate their equivalent resistance, R_7 . (Note that two equal resistances in parallel have an equivalent resistance of half each other's value.)

$$R_7 = 30\ \Omega$$

R_7 and R_1 combine in series to give the total circuit resistance:

$$R_T = 30\ \Omega + 30\ \Omega = 60\ \Omega$$

Now calculate total circuit current:

Knowing E_T and R_T , use Ohm's law to calculate I_T .

$$I_T = 240 \text{ V} \div 60 \Omega = 4 \text{ A}$$

Calculate branch currents and voltages across the resistors R_1 and R_5 :

Use I_T and R_1 in Ohm's law to find E_1 :

$$E_1 = 4 \text{ A} \times 30 \Omega = 120 \text{ V}$$

Use E_1 and E_T with Kirchhoff's voltage law to calculate E_5 :

$$\begin{aligned} E_5 &= E_T - E_1 \\ &= 240 \text{ V} - 120 \text{ V} = 120 \text{ V} \end{aligned}$$

Knowing E_5 and R_5 , use Ohm's law to find I_5 :

$$I_5 = 120 \text{ V} \div 60 \Omega = 2 \text{ A}$$

Calculate branch currents and voltages across the resistors R_1 , R_3 and R_4 :

Rather than considering the individual resistances R_2 , R_3 and R_4 , it is easier at this point to refer to E_6 , the voltage across their equivalent resistance, R_6 , and the current I_6 through it.

Calculate I_6 using I_5 and I_T with Kirchhoff's law:

$$I_6 = 4 \text{ A} - 2 \text{ A} = 2 \text{ A}$$

Now look at I_6 and E_6 in some detail. It may be helpful to look at Figure 15. It is a subcircuit containing R_2 , R_3 and R_4 .

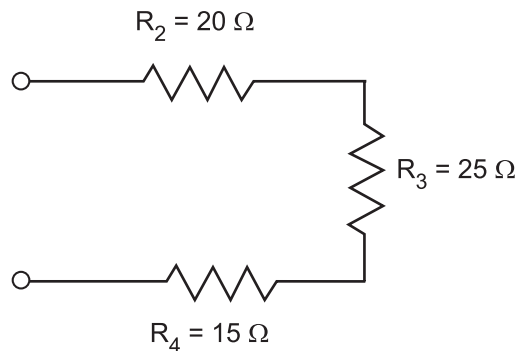


Figure 15—Partial circuit for Example 6

The subcircuit is a series circuit with a 120 V source voltage, a 2 A current, and 20 Ω , 25 Ω and 15 Ω resistors.

In this subcircuit, $I_2 = I_3 = I_4 = 2 \text{ A}$.

Use these current values and the resistance values with Ohm's law to calculate E_2 , E_3 and E_4 :

$$E_2 = 2 \text{ A} \times 20 \Omega = 40 \text{ V}$$

$$E_3 = 2 \text{ A} \times 25 \Omega = 50 \text{ V}$$

$$E_4 = 2 \text{ A} \times 15 \Omega = 30 \text{ V}$$

The calculated values are shown in Table 5:

Table 5 Calculated and given circuit values for Example 6

	E	I	R
Resistor 1	(120 V)	(4 A)	30 Ω
Resistor 2	(40 V)	(2 A)	20 Ω
Resistor 3	(50 V)	(2 A)	25 Ω
Resistor 4	(30 V)	(2 A)	15 Ω
Resistor 5	(120 V)	(2 A)	60 Ω
Total	240 V	(4 A)	(60 Ω)

Example 7: The four resistive circuits in Figures 16a, 16b, 16c and 16d are equivalent to each other. They illustrate the steps needed to determine the total effective resistance in the circuit, R_T . Using all the information given in Figure 16, calculate the following:

$$E_1, E_2, E_3, E_4, E_5$$

$$I_1, I_2, I_3, I_4, I_5 \text{ and } I_T$$

$$\text{Assume } E_T = 300 \text{ V.}$$

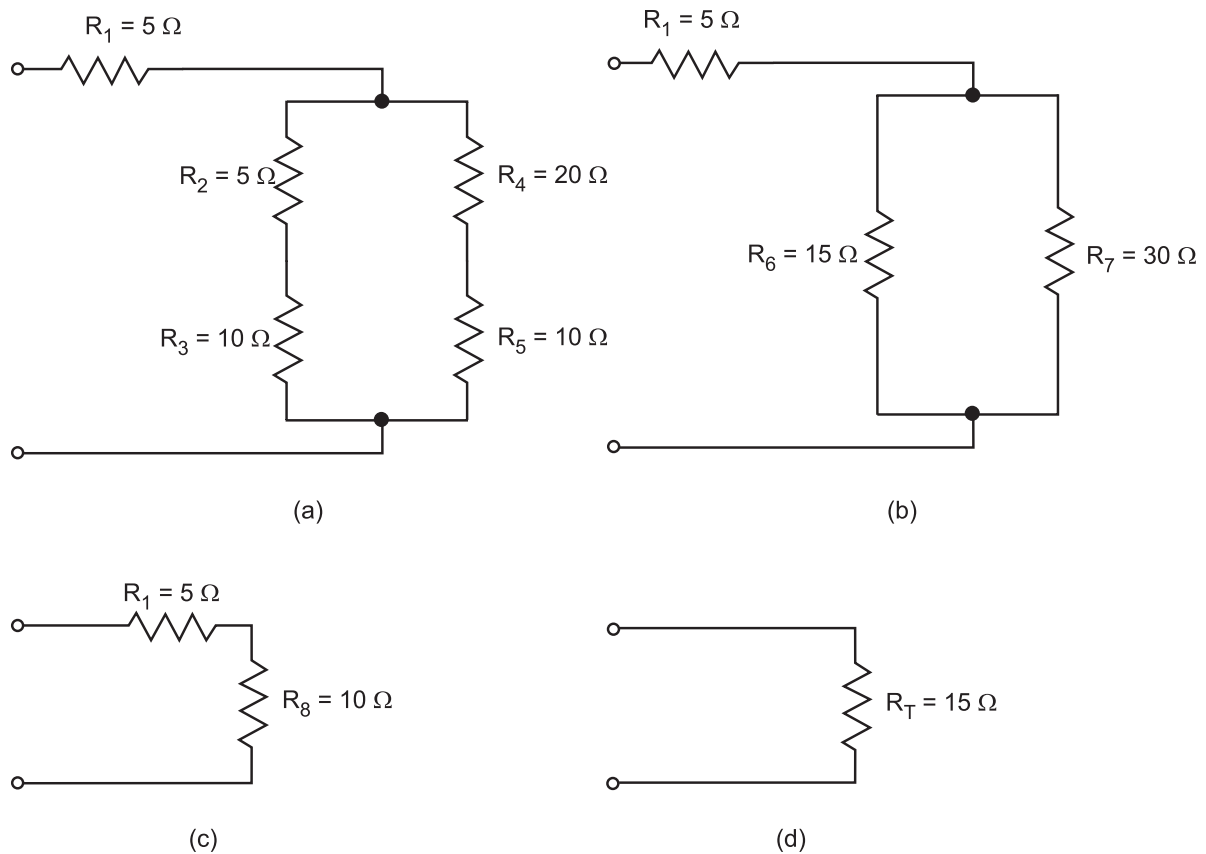


Figure 16—Equivalent resistive circuits for Example 7

Calculate and use total current:

Since you know E_T and R_T , use Ohm's law to find I_T :

$$I_T = 300 \text{ V} \div 15 \Omega = 20 \text{ A}$$

Since R_1 is connected to the main circuit in series, the total current flows through it:

$$I_1 = I_T = 20 \text{ A}$$

Use I_1 and R_1 with Ohm's law to find the voltage drop across R_1 :

$$E_1 = 20 \text{ A} \times 5 \Omega = 100 \text{ V}$$

Calculate voltages across the resistors:

To calculate the voltage values across R_2 , R_3 , R_4 and R_5 , you must know the branch currents through them. To obtain these, first find the voltage across the two main branches.

By Kirchhoff's law, total voltage across the branches containing R_6 and R_7 must be equal to the source voltage less the voltage drop at R_1 .

$$\begin{aligned} E_6 = E_7 &= E_T - E_1 \\ &= 300 \text{ V} - 100 \text{ V} = 200 \text{ V} \end{aligned}$$

Also, Figure 16c shows that the total current flows through equivalent resistance R_8 ; so calculating the voltage E_8 across R_8 from Ohm's law:

$$E_8 = 20 \text{ A} \times 10 \Omega = 200 \text{ V}$$

and because $R_6 // R_7$,

$$E_8 = E_6 = E_7 = 200 \text{ V}$$

Now calculate branch currents I_6 and I_7 . Once these are known, you can determine the voltage drops E_2 , E_3 , E_4 and E_5 .

Now calculate I_6 and I_7 using Ohm's law:

$$\begin{aligned} I_6 &= E_6 \div R_6 \\ &= 200 \text{ V} \div 15 \Omega = 13.33 \text{ A} \end{aligned}$$

and similarly,

$$I_7 = 200 \text{ V} \div 30 \Omega = 6.67 \text{ A}$$

R_2 and R_3 are in series, so that

$$I_2 = I_3 = I_6 = 13.33 \text{ A}$$

and R_4 and R_5 are in series, so that

$$I_4 = I_5 = I_7 = 6.67 \text{ A}.$$

Now, use these current values with Ohm's law to calculate the remaining required voltage values:

$$E_2 = 13.33 \text{ A} \times 5 \Omega = 66.65 \text{ V}$$

$$E_3 = 13.33 \text{ A} \times 10 \Omega = 133.3 \text{ V}$$

$$E_4 = 6.67 \text{ A} \times 20 \Omega = 133.4 \text{ V}$$

$$E_5 = 6.67 \text{ A} \times 10 \Omega = 66.70 \text{ V}$$

This completes all the required calculations.



Now do Self-Test 8 and check your answers.

Self-Test 8

1. For the circuit shown in Figure 1, calculate the following:

a. total circuit resistance _____

b. total circuit current _____

c. voltage drop across each of the resistors _____

d. total power developed in the circuit. _____

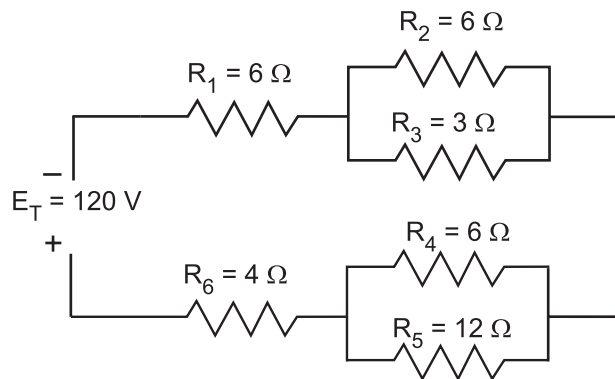


Figure 1—Circuit for Question 1

2. Using the circuit shown in Figure 2, fill in the missing values in Table 1.

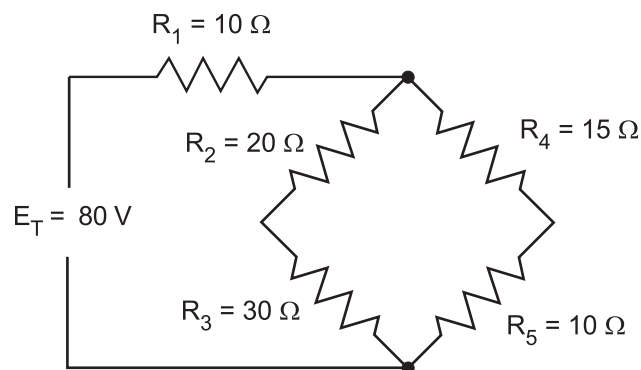


Figure 2—Circuit for Question 2

Table 1 Circuit value chart for Question 2

	E	I	R
Resistor 1			10 Ω
Resistor 2			20 Ω
Resistor 3			30 Ω
Resistor 4			15 Ω
Resistor 5			10 Ω
Total	80 V		

3. Six equal resistors connected in series produce total circuit resistance of 72 ohms. If these same six resistors are connected in parallel, what is the total circuit resistance?

4. From the information shown in Figure 3, determine the following:

- a. E_T _____
- b. I_3 _____
- c. R_1 _____
- d. R_2 _____
- e. R_3 _____

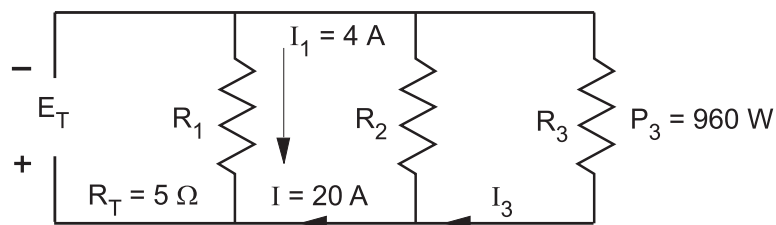


Figure 3—Circuit for Question 4

5. From the information shown in Figure 4, determine the reading expected on each of the six voltmeter connections.

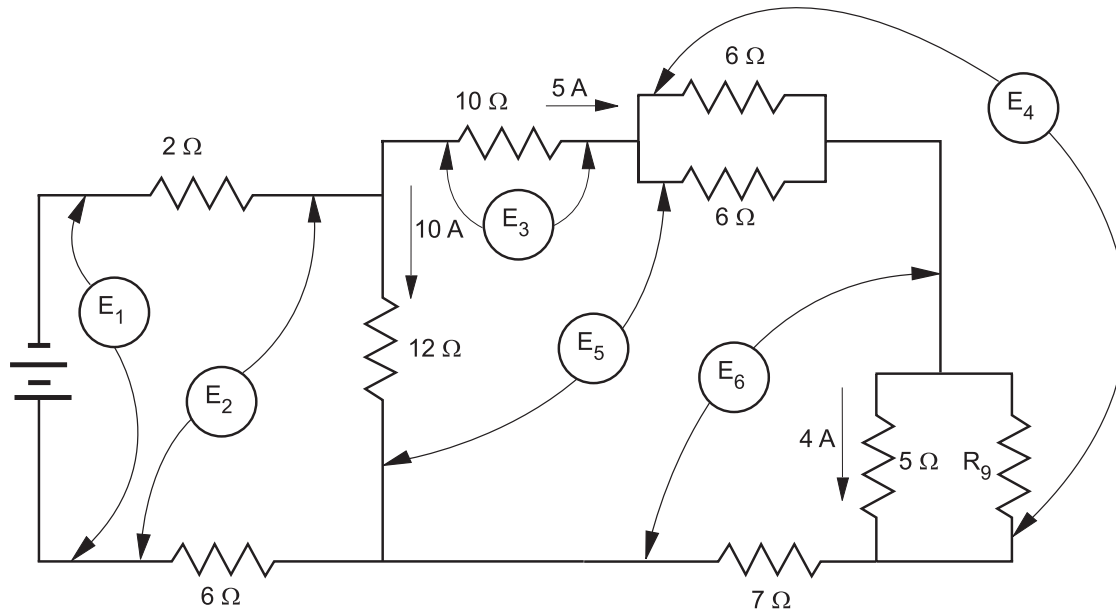


Figure 4—Circuit showing voltmeter connections for Question 5

_____	_____
_____	_____
_____	_____

6. For the circuit shown in Figure 5, what is the expected resistance reading if an ohmmeter is connected across the following points?

a. A and B

b. B and C

c. A and C

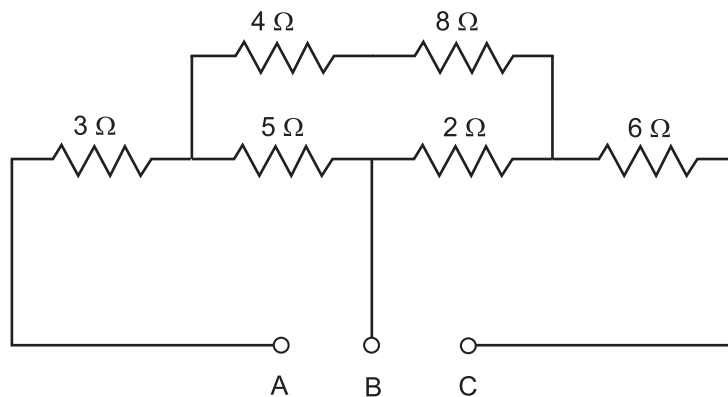


Figure 5—Circuit for Question 6

7. Using the values given in Figure 6, determine the:

a. supply voltage E_T

b. total circuit current I_T

c. total circuit resistance R_T

d. resistance value R_3

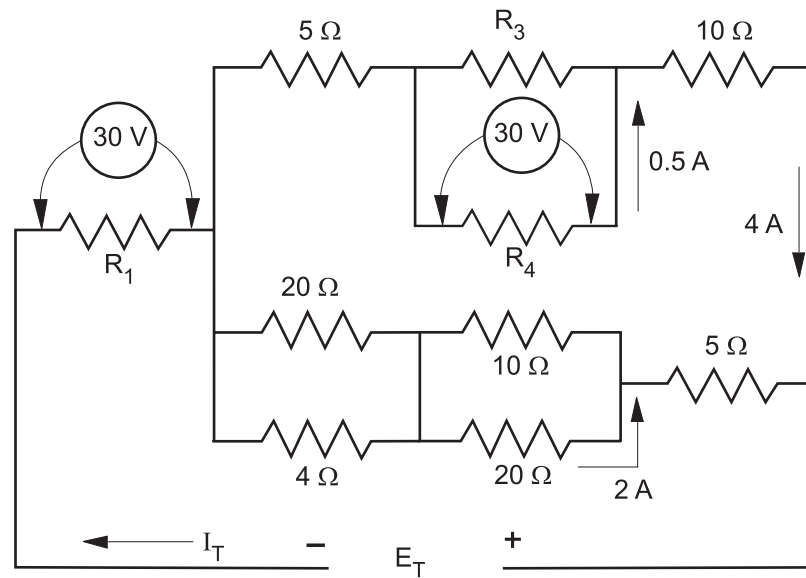


Figure 6—Circuit for Question 7

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 9:

Describe the characteristics of a voltage divider circuit

DC electronic circuits quite often need different voltage values. To get several values of voltage from a common supply, you may use a circuit called a voltage divider.

Construction of a voltage divider circuit

A simple voltage divider is constructed by using series-connected resistors. A voltage divider uses the principle of *proportional voltage drops* in a series circuit. Figure 1 shows three resistors connected in series to a 240 V DC supply. The comparative voltage drops across the resistors are proportional to the individual values of resistance, as shown.

Note: The circuit diagrams in this Learning Task use "K" to symbolize $k\Omega$, in keeping with trade language.

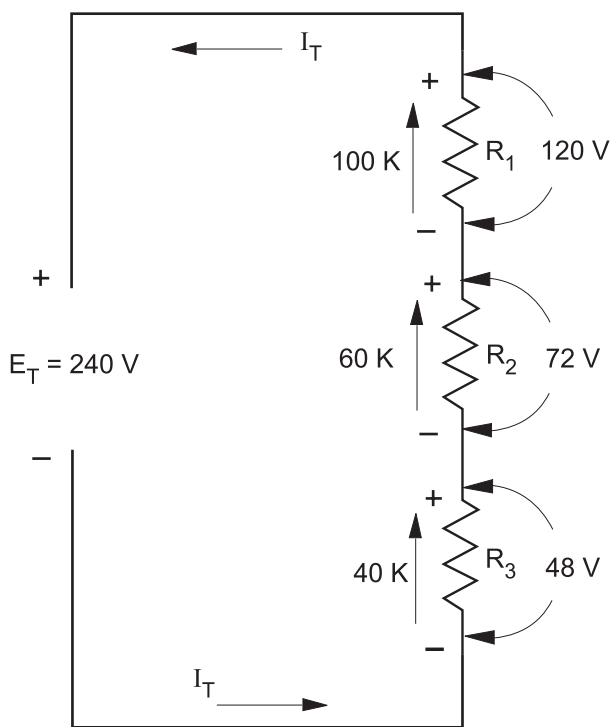


Figure 1—Series resistors

Any of these individual voltage drops could act as a *source* for providing different levels of voltage for another load connected in parallel.

For example, in Figure 1, a load resistor connected in parallel with:

- R_1 is provided with 120 V
- R_2 is provided with 72 V
- R_3 is provided with 48 V

Notice that such a load connected in parallel would also have the “same polarity” as that source resistor.

More commonly, the voltage outputs are measured to a common point. For example, in Figure 2, D is negative and each of the other points (C, B, A) is increasingly more positive.

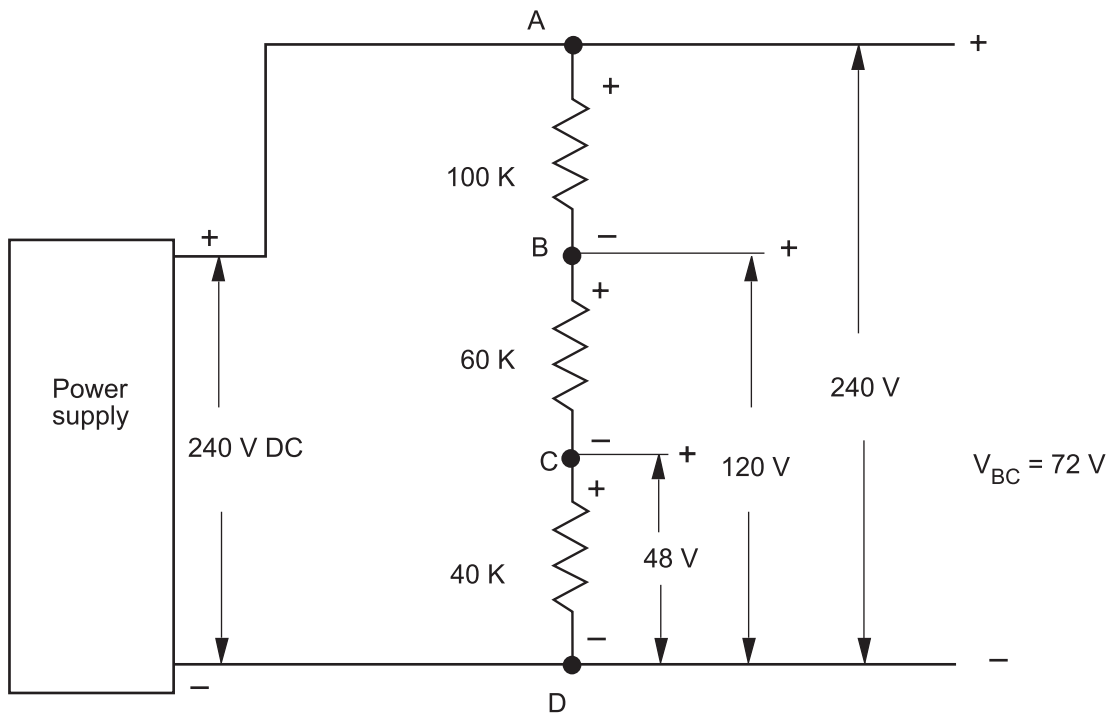


Figure 2—Voltage divider circuit with common point D

Similarly, the common point could be point A, with each of the other points (B, C and D) increasingly more negative, as shown in Figure 3.

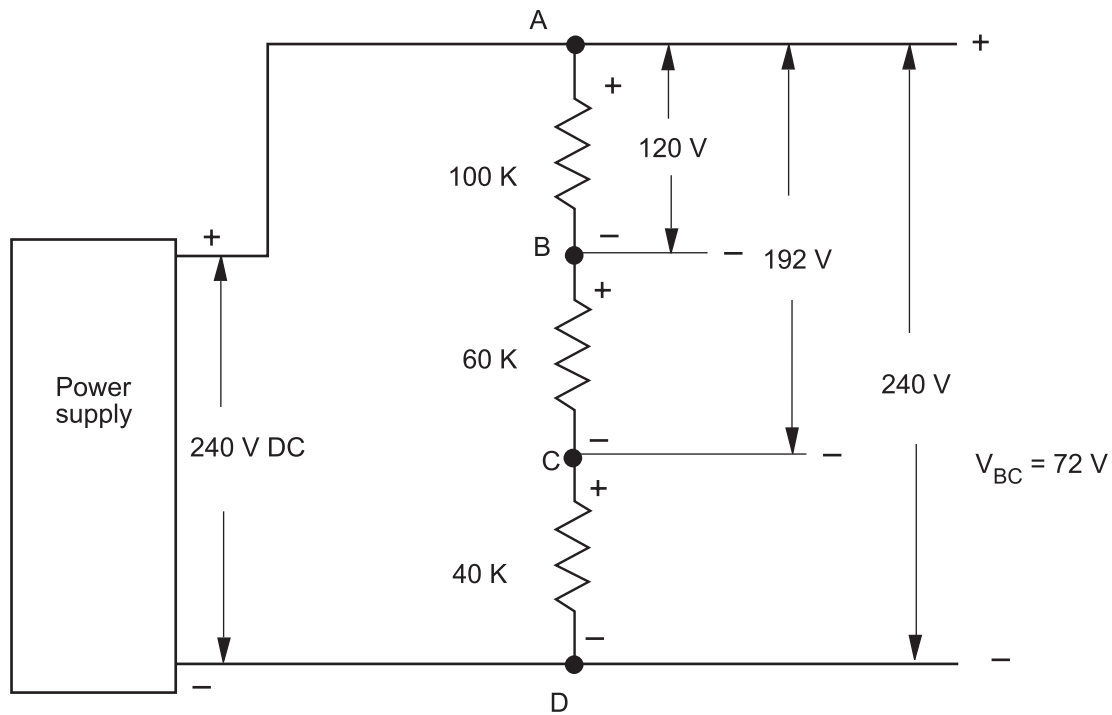


Figure 3—Voltage divider circuit with common point A

Note:

- In Figure 2, point B is considered positive with respect to point D.
- In Figure 3, point B is considered negative with respect to point A.

You should already be aware that polarity is always relative between two points where a difference in potential exists.

Sometimes the *common point* of a power supply is symbolized as *grounded*. In Figure 4, point D is grounded and the remaining points (A, B and C) are marked as positive (with respect to this common point).

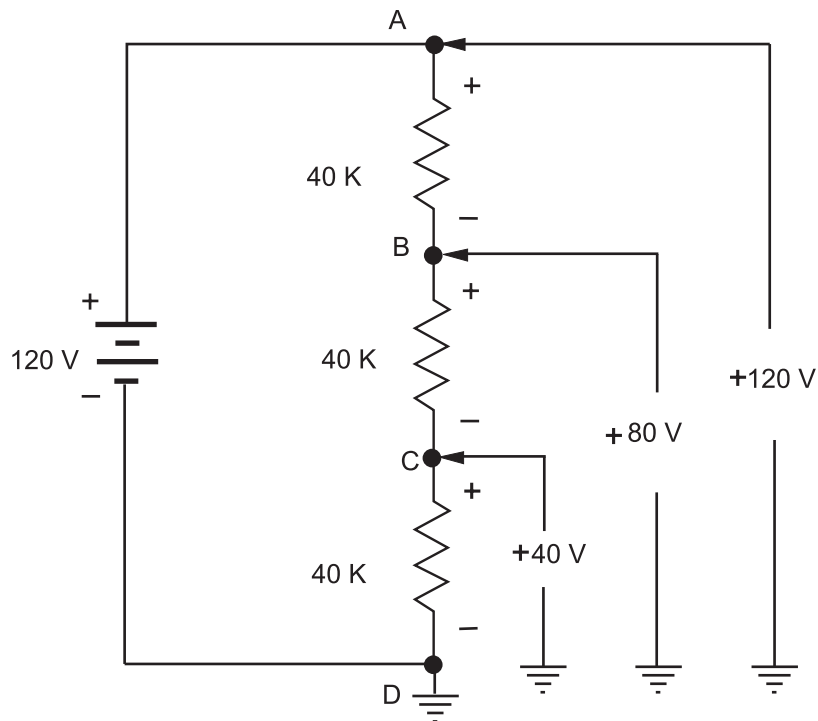


Figure 4—Positive marked voltages with respect to common grounded point

Similarly, if point A is grounded, then the remaining points (B, C and D) are all considered negative with respect to this common point. See Figure 5.

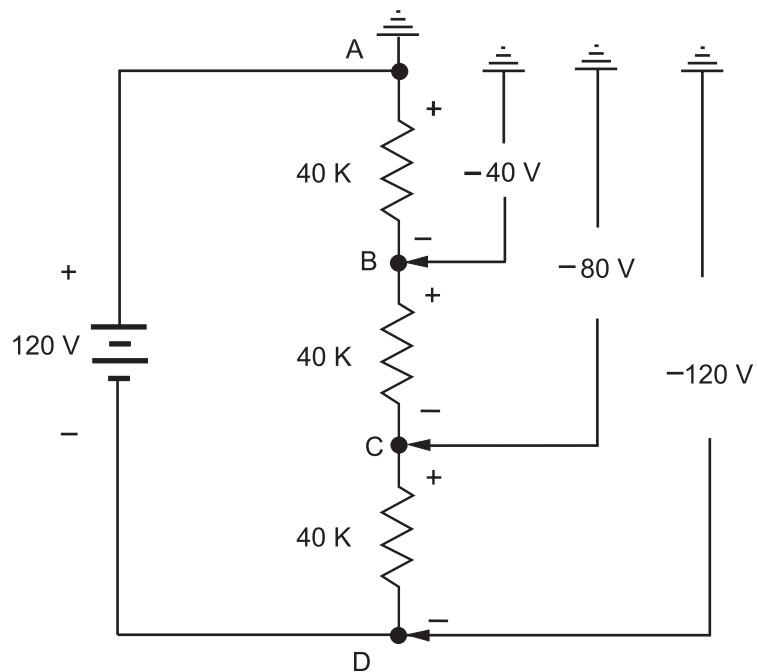


Figure 5—Negative marked voltages with respect to common grounded point

Some power supplies provide both positive and negative potentials with respect to a common point. Figure 6 shows three equal resistors connected in series across 120 V. Individually across each resistor, you measure a 40 V drop.

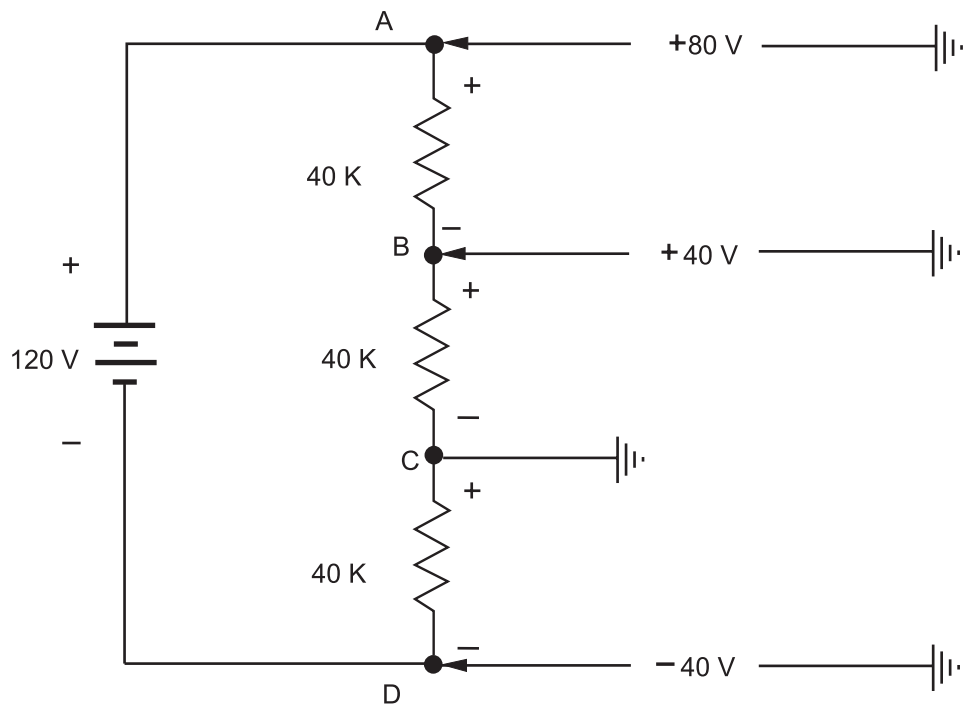


Figure 6—Positive and negative potentials with respect to a common point

By making point C common, both positive and negative voltages can be measured to this common point. Typically, the *common* of power supplies is marked 0 V, and all other voltage levels from the divider circuit are indicated as positive or negative values to this reference.

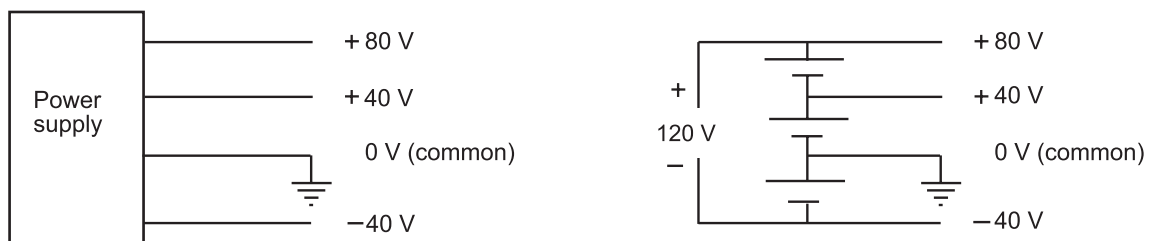


Figure 7—Typical markings on voltage divider circuit

The potentiometer

A potentiometer is a variable resistor with three terminals. It is commonly used in electronic circuits for varying voltage. When a 500 kW potentiometer is connected as shown in Figure 8, it can act as a variable voltage divider.

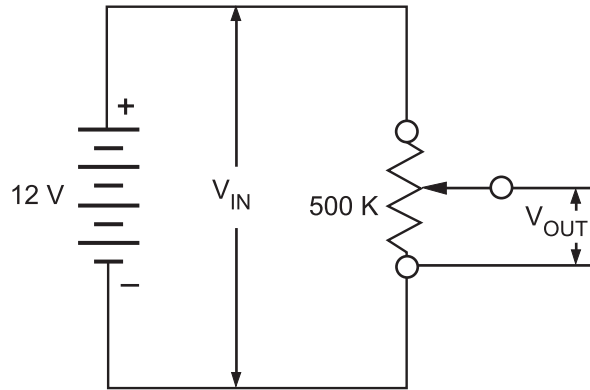


Figure 8—Potentiometer circuit

- With the slider adjusted midway, the output voltage is one-half of the input voltage.
- If the slider arm is turned clockwise (upward in Figure 8), the output voltage increases.
- If the slider is turned counter-clockwise (downward in Figure 8), the output voltage decreases.



Now do Self-Test 9 and check your answers.

Self-Test 9

1. A voltage divider is constructed with three resistors as shown in Figure 1. With no load connected, what are the polarity and voltage level expected with respect to ground, at the following points:

a. A

b. B

c. C

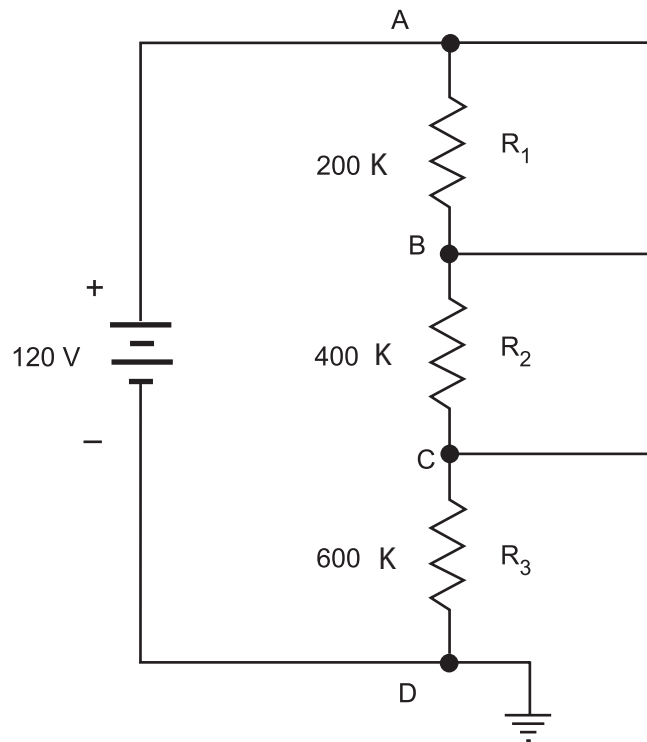


Figure 1—Circuit for Question 1

2. If the ground (or common) is moved from point D to point C in Figure 1, what polarity and voltage level with respect to ground are expected at the following points:

a. A

b. B

c. C

3. A 100 K potentiometer is to be used to provide a variable voltage output, as illustrated in Figure 2. With no load connected, determine the voltage output when the slider is positioned as follows:
- a. in the mid-resistance position _____
 - b. fully counter-clockwise _____
 - c. fully clockwise _____

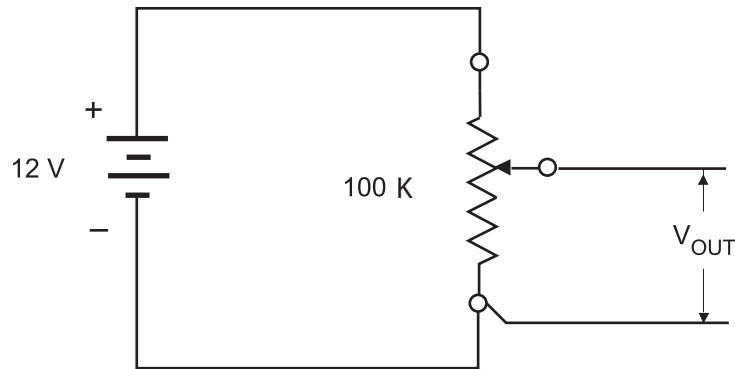


Figure 2—Circuit for Questions 3 and 4

4. The preceding potentiometer is placed in the mid-resistance position, and a 50 K load resistor is connected to the output terminals. What voltage is expected across the load resistor?
- _____

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 10:

Solve problems involving voltage divider circuits

Voltage divider circuits are commonly used with loads that require very low current values—in other words, very high resistance loads. The reason for this is that low resistance loads would cause the net resistance of the divider circuit to change, which would result in voltage values different from those expected. This change in voltage with the amount of connected load is referred to as *voltage regulation*.

Example 1: The unloaded voltage divider

Consider the simple voltage divider circuit shown in Figure 1.

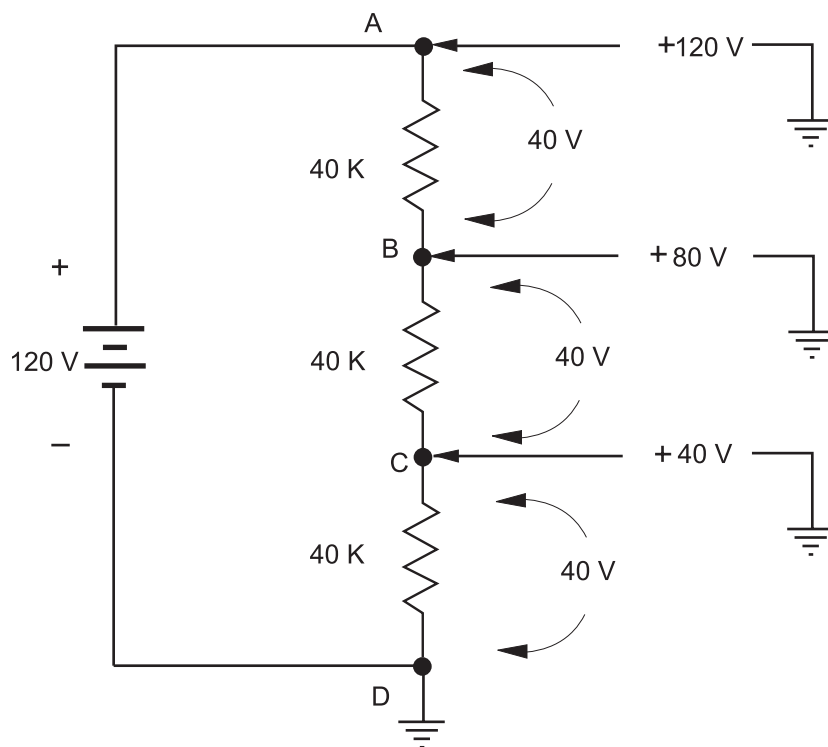


Figure 1—Unloaded voltage divider

A voltage divider is a series circuit. You have already learned that individual IR (voltage) drops in a series circuit are proportional to the series resistance values. The voltage drop across each resistor can be determined by the formula:

$$V_R = \frac{R}{R_T} \times V_T$$

Where:

R = the individual resistance value

R_T = the total series circuit resistance value

V_T = the total circuit voltage

In Figure 1, since all three series-connected resistors are of equal value, the voltage drop across each resistor is calculated as:

$$V_R = \frac{40 \text{ k}}{120 \text{ k}} \times 120 \text{ V} = 40 \text{ V}$$

The voltage from point C to common (point D) = +40 V (since point C is “more positive” than point D).

The voltage from point B to common (point D) = 40 V + 40 V = +80 V.

Similarly, the voltage from point A to common (point D)

= 40 V + 40 V + 40 V = +120 V.

Example 2: The loaded voltage divider (Figure 2)

If a 40 k Ω load resistor (R_L) is connected across points C and D, then the net resistance between points C and D becomes 20 k Ω (the value determined by connecting two 40 k Ω resistors in parallel). This now reduces the total series equivalent resistance to 40 k Ω + 40 k Ω + 20 k Ω = 100 k Ω .

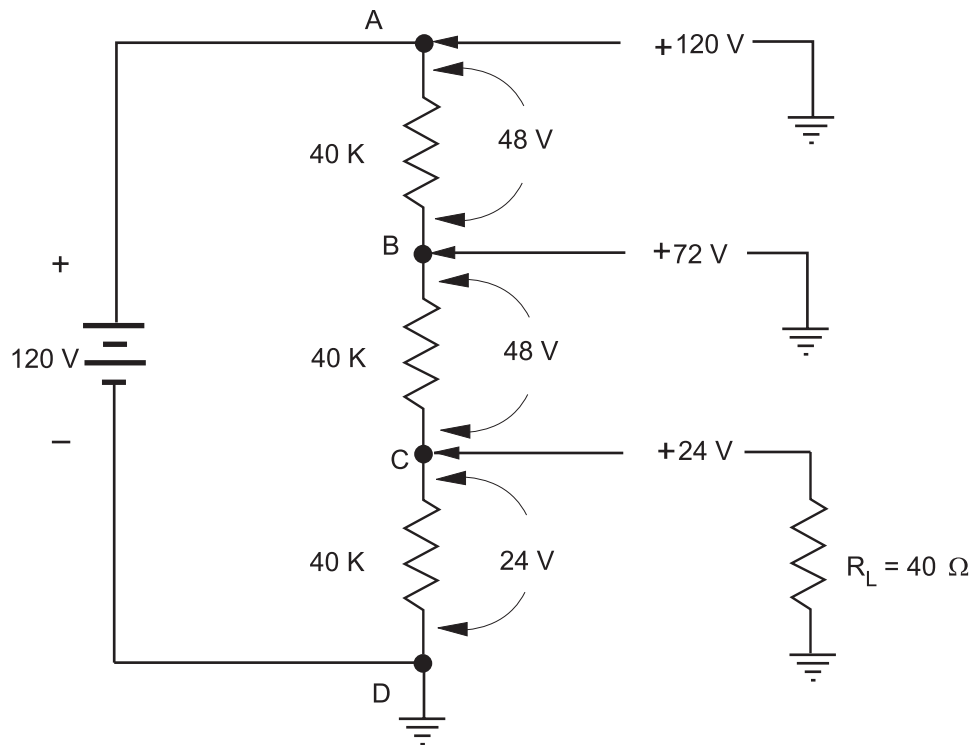


Figure 2—Loaded voltage divider

Now, the voltage from point C to common (point D) = $\frac{20 \text{ k}}{100 \text{ k}} \times 120 \text{ V} = +24 \text{ V}$.

Similarly, the voltage from point B to point C = $\frac{40 \text{ k}}{100 \text{ k}} \times 120 \text{ V} = 48 \text{ V}$ and the voltage from point B to common (point D) = $48 \text{ V} + 24 \text{ V} = +72 \text{ V}$.

Likewise, the voltage from point A to point B = $\frac{40 \text{ k}}{100 \text{ k}} \times 120 \text{ V} = 48 \text{ V}$ and the voltage from point A to common (point D) = $48 \text{ V} + 48 \text{ V} + 24 \text{ V} = +120 \text{ V}$.

The preceding examples show how the voltage output from a simple resistance divider network can change when a load is connected. Voltage regulation must be considered in the practical design of any voltage divider circuit. You will learn more about this in future studies.



Now do Self-Test 10 and check your answers.

Self-Test 10

As shown in Figure 1, a voltage divider draws 2 mA from a DC power supply.

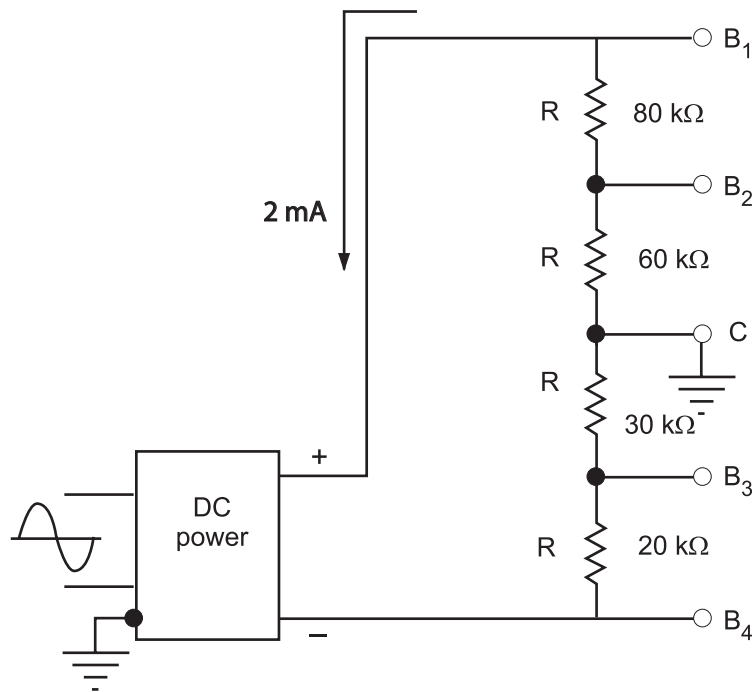


Figure 1—Voltage divider circuit

Determine:

1. The polarity of the output voltages (at each point B) with respect to the common (point C).

2. The “unloaded” value of output voltage (at each point B) with respect to the common (point C).

3. The value of output voltage (at each point B) with respect to the common point if a 120 kΩ load resistor is connected across terminals B₂ and common.

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 11:

Describe the characteristics of a bridge circuit

To understand the operation of certain circuits and the operation of some instruments, you must know the fundamentals of a bridge circuit. This type of circuit has been utilized for a long time in various forms in the electrical and electronics industry. In order to know how a bridge circuit functions, you must understand what is meant by terms such as potential, positive potential, negative potential and difference in potential.

Bridge circuit

A bridge circuit in its simplest form is illustrated in Figure 1. Two identical resistors, R_1 and R_2 , are connected in parallel across a 120 V source. The network becomes a bridge circuit when a cross-connection, or “bridge,” is placed between the two resistors.

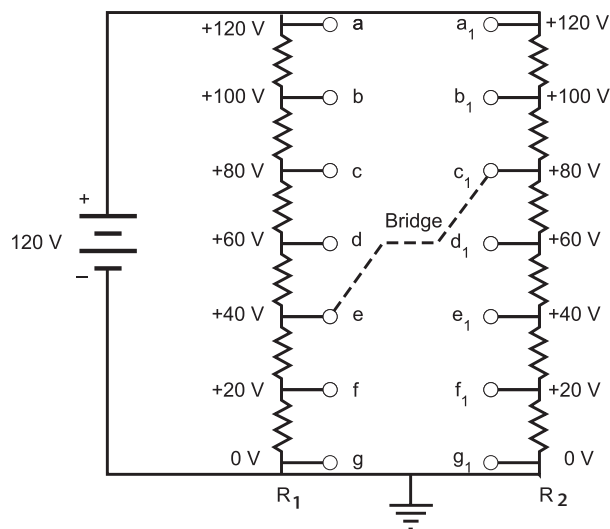


Figure 1—A bridge circuit

Because the two resistors here are identical, voltages are dropped across them at the same rate. Therefore the points identified by the same letters, a – a_1 , b – b_1 , etc., have the same potential (or voltage) with respect to ground. This means they are at the same potential relative to each other. If a bridge is connected across two points of equal potential, no current flows in the bridge.

If the bridge is connected across two points of unequal potential, current does flow. The magnitude of the current depends on the difference in potential between the two points. The direction of the current flow depends on the relative polarities of the two points. In conventional current flow, current flows from the point of higher potential to the point of lower potential. In electron current flow, current flows in the opposite direction; that is, from the point of lower potential to the point of higher potential. Consider the bridge circuits in Figure 2 (a), (b), and (c).

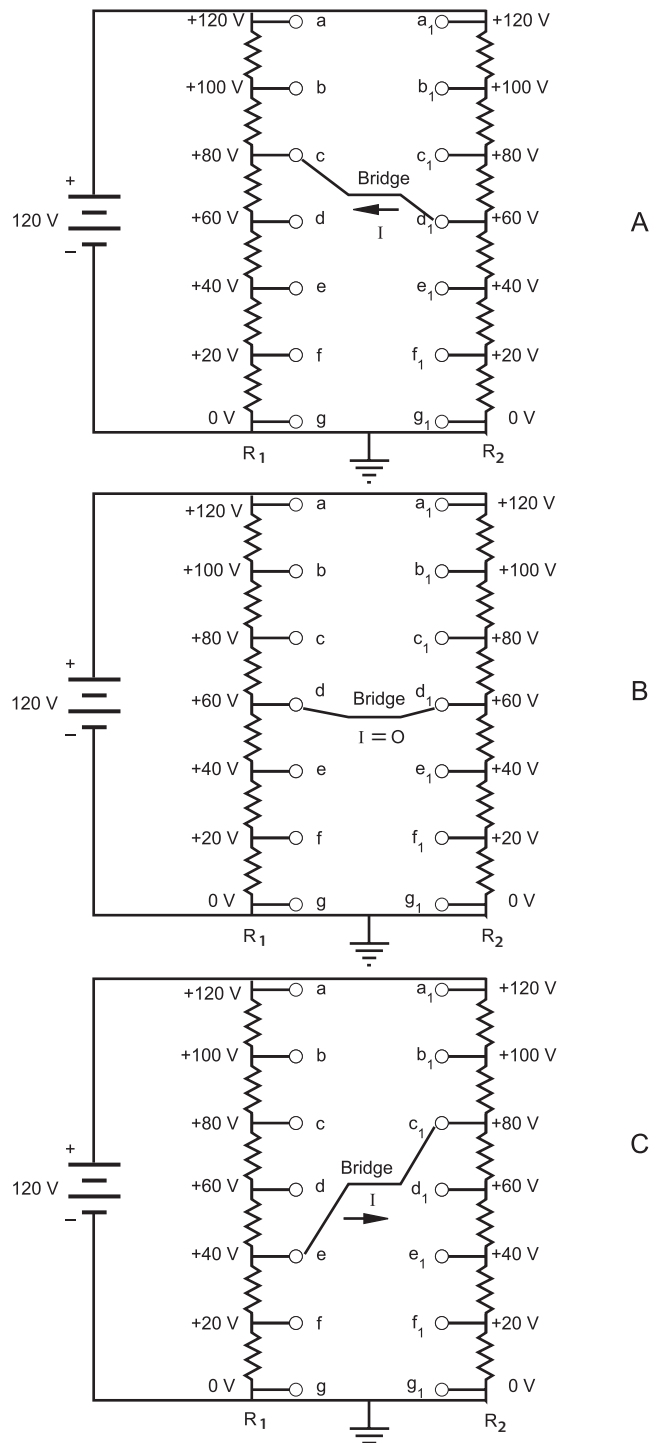


Figure 2—Bridge circuit behaviour

In Figure 2a, current is shown going from right to left because point d_1 is negative with respect to point c. In Figure 2c, current is shown going from left to right because point e is negative with respect to point c_1 .

In Figure 2b, the current is zero because there is no difference in potential between these two points. In Figure 2b, we have a *balanced* bridge condition. In this condition, there is no difference in potential and no current.

In Figures 2a and 2c there is a difference in potential, the bridge is *unbalanced*, and a current detecting device such as a galvanometer would register current.

Wheatstone bridge

The Wheatstone bridge is widely used to make precision resistance measurements. Wheatstone bridge readings are far more precise, and more accurate, than readings obtained with an ohmmeter.

The diagram of a Wheatstone bridge is shown in Figure 3. R_1 , R_2 and R_3 are precision resistors whose values may be altered. R_x is the resistor under test.

After the bridge has been balanced by adjustment of the precision resistors, the unknown resistance can be determined using a simple formula. The precision resistors, galvanometer, switch and battery form an integral part of the instrument. R_x is connected across two external terminals on the instrument. When the bridge is *balanced*, points C and D are at the same potential. The zero-centre galvanometer pointer will not deflect if the switch is closed under this condition. It can be shown mathematically that the cross products of the resistors on opposite sides are equal under this condition.

Thus, in Figure 3: $R_1 \times R_x = R_2 \times R_3$.

Therefore: $R_x = \frac{R_2 \times R_3}{R_1}$.

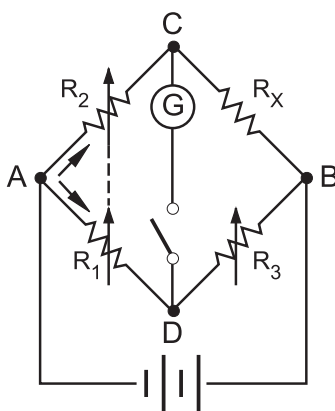


Figure 3—Wheatstone bridge circuit

Applications

The top section of a commercial Wheatstone bridge is shown in Figure 4.

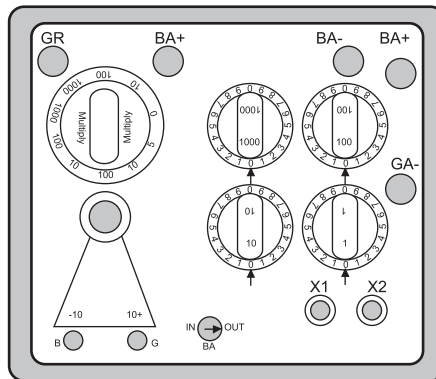


Figure 4—Commercial Wheatstone bridge

The Wheatstone bridge principle is the basis for several other bridge circuits used in industry. Variations of the Wheatstone bridge are used in AC bridges. An AC source replaces the battery. The galvanometer is replaced by another sensing device such as an audio detector.

Two common bridges are the capacitance bridge and the inductance bridge. These are used to measure unknown capacitance and inductance values. The bridge principle is also used in instruments designed to pinpoint fault locations in underground cables.

Figure 5 shows a bridge circuit used to detect changes in temperature. R_3 is a thermistor. The thermistor is a temperature-sensitive resistor that has a negative temperature coefficient of resistance; its resistance will vary by a definite amount for a given change in temperature.

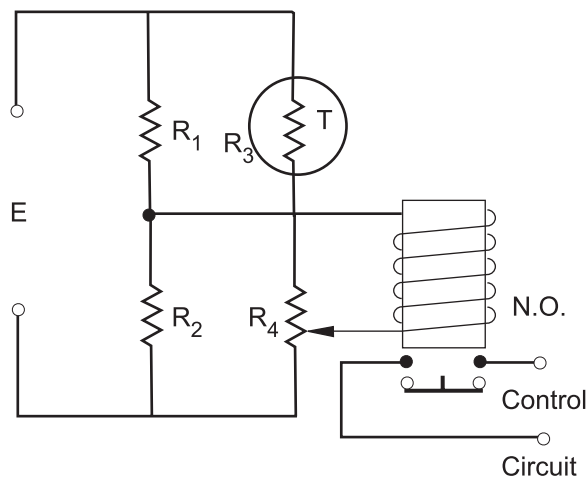


Figure 5—Bridge circuit for temperature control

The bridge can be balanced at any desired temperature value by means of the sliding contact on R_4 . When the bridge is balanced, no current flows in the relay coil. Should the temperature vary either up or down from the preset value, the bridge becomes unbalanced and the relay coil

is energized. This activates the control circuit device, which may turn on an indicating light, set off an alarm, start a motor, or cause any other desired effect.

Circuits could be designed to respond to both the magnitude and direction of the current. For example, a damper motor could rotate in one direction for a rising temperature and in the opposite direction for a falling temperature, causing a shutter to open or close.



Now do Self-Test 11 and check your answers.

Self-Test 11

1. A current does not flow between points that have _____ potential (the same/a difference in).
2. Electron current flows from a point of _____ (higher/lower) potential to a point of _____ (higher/lower).
3. The magnitude of the current flow between points in an electric circuit depends on what feature between the two potentials?

4. What is another name for the term *potential*?

5. In the bridge circuit of Figure 1, what is the voltage across the switch if the voltage to ground is 120 V?

6. If a galvanometer is connected across the switch in Figure 1, electron flow would be from the _____ (left/right) side to the _____ side (left/right).

7. Write the equation for R_x for the bridge circuit in Figure 2 in terms of the other three resistors for the balanced bridge condition.

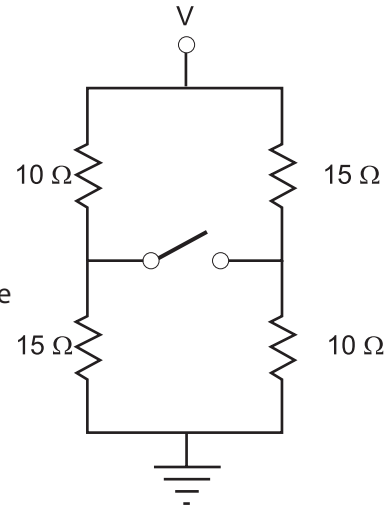


Figure 1

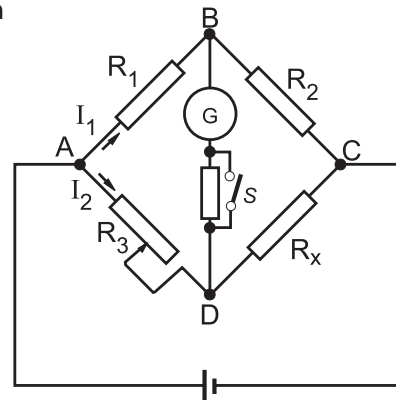


Figure 2

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 12:

Solve problems involving bridge circuits

As stated in Learning Task 1, when the bridge is balanced, the resistor cross component products are equal. To show why this is, we will examine the Wheatstone bridge circuit and analyze the circuit components.

Balanced bridge condition

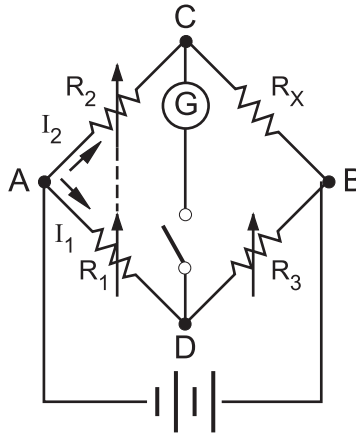


Figure 1—Balanced bridge condition

Referring to Figure 1, battery current enters at point A, and divides as it does in any parallel circuit. I_1 flows through R_1 and R_3 , and I_2 flows through R_2 and R_x . The currents reunite at B and return to the battery. R_1 , R_2 and R_3 have been adjusted so that potentials at C and D are equal and the galvanometer reads zero. Since $E_{CD} = 0$, it means $E_{AC} = E_{AD'}$ and $E_{CB} = E_{DB}$. (Verify this by using Kirchhoff's voltage law around loops ACDA and BCDB.)

Since $E_{AC} = E_{AD'}$ it follows that $I_2 \times R_2 = I_1 \times R_1$ [1]

And because $E_{CB} = E_{DB}$, it follows that $I_2 \times R_x = I_1 \times R_3$ [2]

If products of numbers are equal, the ratios of the products are equal. You could check this by substituting numbers for the letters in [1] and [2].

Then:

$$I_2 \times R_2 = I_1 \times R_1$$

$$I_2 \times R_x = I_1 \times R_3$$

So:

$$R_1 \times R_x = R_2 \times R_3 \text{ (cross products are equal)}$$

$$\text{and: } R_x = \frac{R_2}{R_1} \times R_3$$

Here R_1 and R_2 are called the ratio arms. They change together and maintain ratios in increments of 10. This combination is called the *multiplier*, and it may be 0.001, 0.01, 0.1, 1, 10, 100 or 1000. R_3 is independent and it may be varied from 0 to 9999 Ω .

Thus, when the bridge is balanced, $R_x = R_3$ times the ratio R_2/R_1 .

Example 1: Find R_x in Figure 2 if the bridge is balanced when $R_1 = 100 \Omega$, $R_2 = 10 \Omega$ and $R_3 = 127 \Omega$.

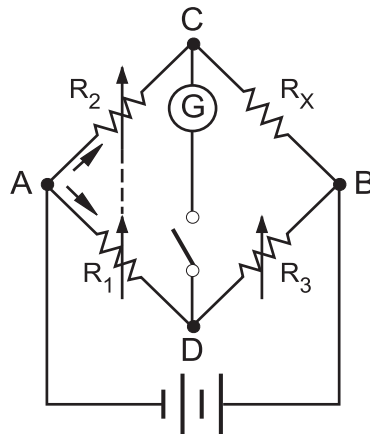


Figure 2—Bridge balancing

Solution:

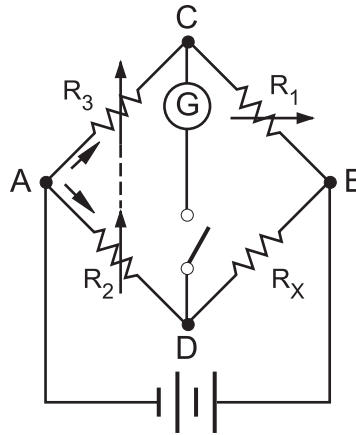
$$R_x = \frac{R_2}{R_1} \times R_3 = \frac{10}{100} \times 127 \Omega = 12.7 \Omega$$

Example 2: Find R_x if $R_2/R_1 = 0.001$, and $R_3 = 1941 \Omega$ in Figure 2.

Solution:

$$R_x = \frac{R_2 \times R_3}{R_1} = 0.001 \times 1941 = 1.941 \Omega$$

While in practice the multiplier ratios will be those stated above, questions are frequently asked using any values of R_1 , R_2 and R_3 . And, these could be shown on any resistors.

Figure 3—Value of R_x

Example 3: Find the value of R_x in Figure 3 if $R_1 = 120\ \Omega$, $R_2 = 420\ \Omega$ and $R_3 = 36\ \Omega$.

Solution:

$$R_x = \frac{R_2}{R_3} \times R_1 = \frac{420}{36} \times 120\ \Omega = 1400\ \Omega$$



Now do Self-Test 12 and check your answers.

Self-Test 12

- The Wheatstone bridge is most commonly used for measuring
 - current
 - voltage
 - high resistance
 - precise resistance

- What is the current flow in the galvanometer when the bridge is balanced?

- What is the purpose of switch S_1 in Figure 1?

- For the bridge in Figure 1, write the equation for the unknown resistance in terms of the other resistors.

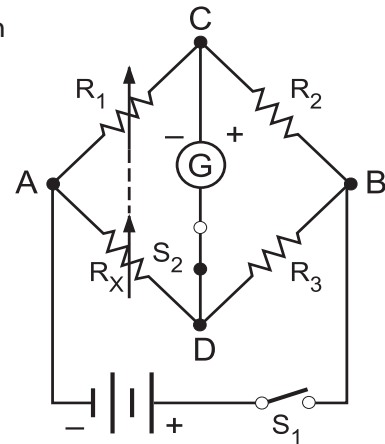


Figure 1—Wheatstone bridge

- If the bridge is balanced when the ratio arms are at 0.01 multiplier and the resistance is $117\ \Omega$, what is the value of R_x ?

- In Figure 1, find the value of R_x if $R_1 = 1000$, $R_2 = 100$ and $R_3 = 256\ \Omega$.

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 13:

Describe the features of a three-wire distribution system

Electrical energy to most individual and small commercial buildings is distributed through a 120 V/240 V, single-phase, three-wire system.

Several advantages are gained by using this method of distribution:

- Copper conductor ampacity requirements can be reduced.
- Two different voltages (120 V and 240 V) are available.
- Conductor line drop (volts) and line loss (watts) are minimized.
- Improved safety is established through grounding the neutral.

Source connections

Although alternating current (AC) is more commonly used, the basic concepts of a three-wire circuit can be more easily identified by using direct current (DC).



Although AC reverses in polarity at equal time intervals, circuit analysis is often done by assuming an instantaneous polarity.

A three-wire circuit is accomplished by connecting two 120 V sources in a *series-aiding configuration*, as in Figure 1. The conductor taken from the common point between the two sources is called the *neutral* conductor. Conductors taken from the two outer points are called the *line* or *hot* conductors.

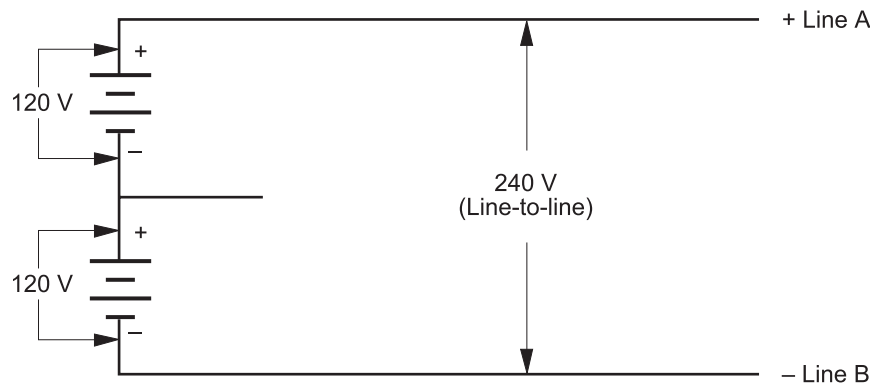


Figure 1—Three-wire circuit

- Voltage measured *line-to-line* is 240 V.
- Voltage measured from either-line-to-neutral is 120 V.

This allows connection to either 120 V loads (such as lighting) or 240 V loads (such as ranges or clothes dryers). See Figure 2.



The 120 V and 240 V referred to are the standard values. But sometimes values of 110 V and 220 V, or 115 V and 230 V are used in three-wire circuits

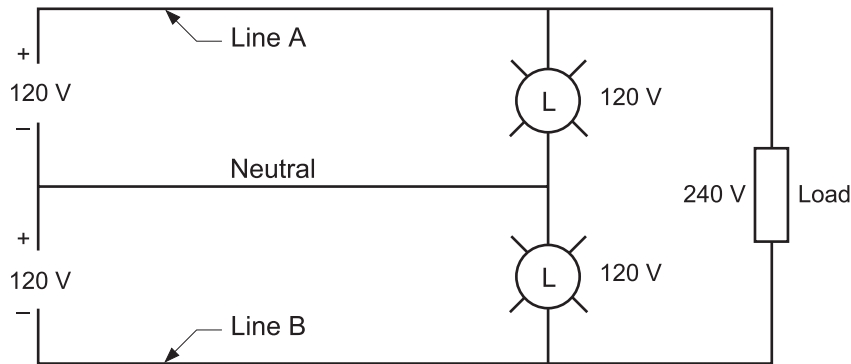


Figure 2—Three-wire distribution

For safety reasons, the Canadian Electrical Code is explicit as to how circuit conductors are identified by colour:

- Insulation on the two line conductors is usually black (or sometimes one is black and one is red).
- The neutral always has white insulation.

The neutral is also grounded (directly connected to earth) at the source of supply.

Grounding the neutral conductor

The earth we stand on is a conductor of electricity. So to reduce the hazards of electrical shock and improve safety, electrical distribution systems usually have one of the circuit conductors connected to earth, or as we say, *grounded*.

In most electrical systems, the neutral conductor is grounded at the supply by directly connecting it to earth by another conductor (called the system grounding conductor) or by an electrode. Although grounding electrical distribution systems is an elaborate topic, consider the simplified example that follows.

The fundamental purpose of grounding, as defined by the Canadian Electrical Code, is to guard against electrical shocks and fire hazards. But what makes a grounded electrical system safer? Let's look at an ungrounded, 120V/240 V wiring system with a fault, as shown in Figure 3. Theoretically, if an insulation fault occurs at a piece of equipment (so that a line conductor makes accidental contact with the metal frame), nothing should happen. However, if an accidental ground should occur, as in Figure 3, and you come in contact with the faulty equipment and ground, you will experience a shock of 240 V (the line-to-line voltage.)

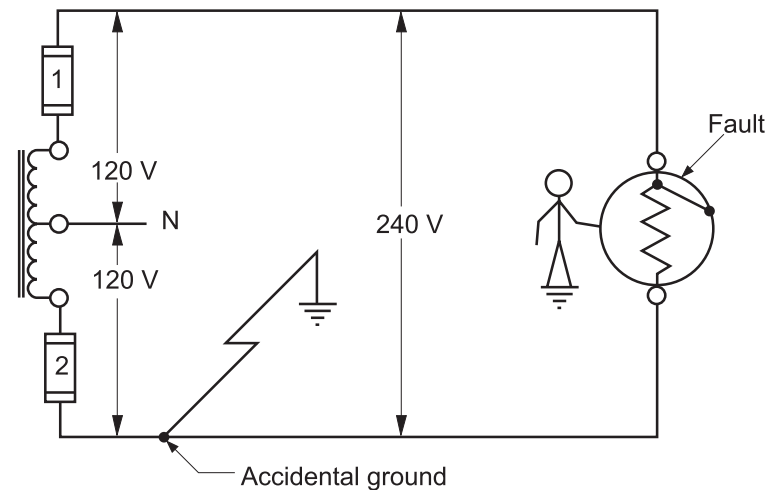


Figure 3—Accidental fault in an ungrounded wiring system

Now let's look at the same wiring system with the neutral purposely grounded, as shown in Figure 4.

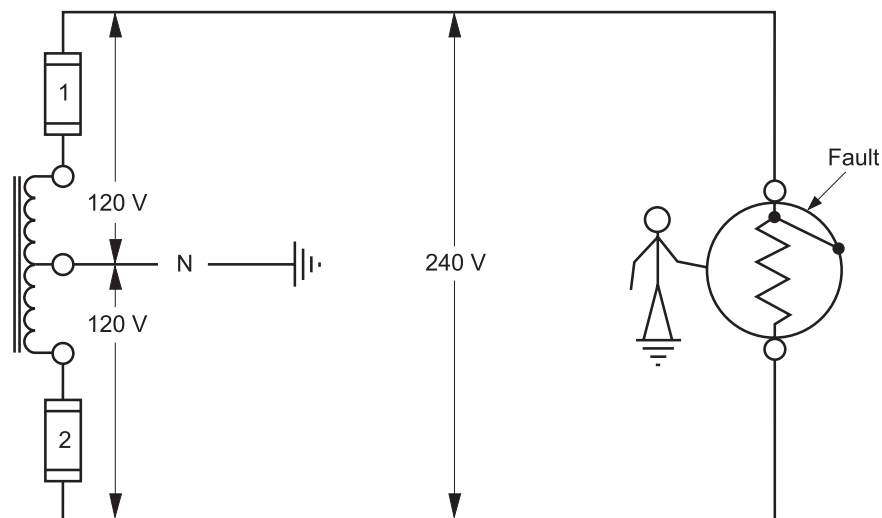


Figure 4—Accidental fault in a grounded neutral wiring system

With the neutral grounded and the same equipment fault as previously described, the person coming in contact with both the metal frame of the equipment and the earth now experiences a shock of only 120 V (which is the line-to-neutral voltage). The shock voltage has been reduced by 50%.

To further minimize shock hazard, we not only ground the wiring system but we also ground all metallic, non-current-carrying parts of equipment by using a *bonding conductor*. This is an important step if maximum safety is to be achieved.

As shown in Figure 5, if the equipment is also grounded intentionally, then a line-to-frame fault condition offers a low resistance path for current flow back to the system neutral. Essentially

this is a line-to-neutral short circuit, which causes the circuit overcurrent device to trip, thus eliminating the shock hazard from the system.

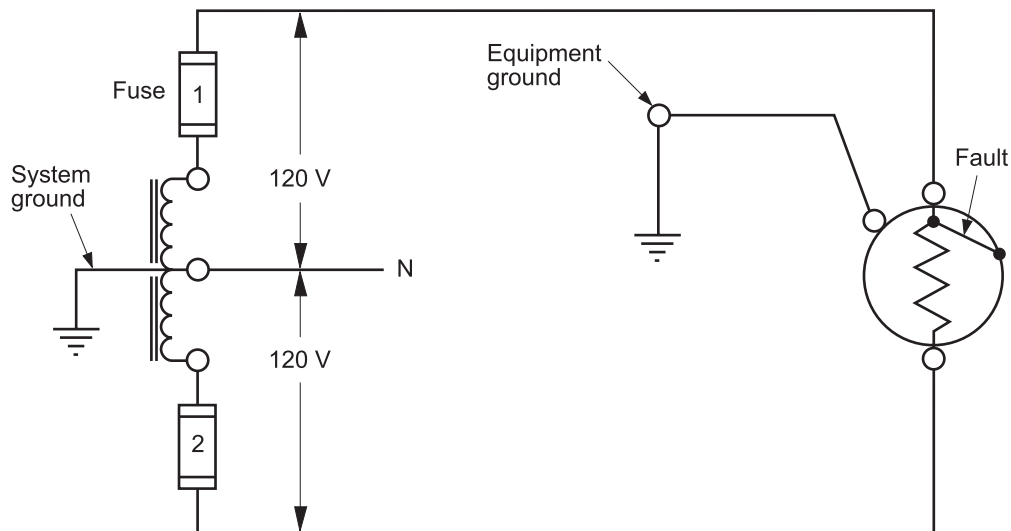


Figure 5—A properly grounded wiring system



Although properly grounded wiring systems do not eliminate shock hazard, they certainly cut the odds!

Balanced three-wire circuits

When an equal amount of load resistance is connected across each of the 120 V line-to-neutral sections of a three-wire circuit, we have what is called a *balanced load* condition. This is demonstrated by the circuit shown in Figure 6.

If each of the 120 V lamps in Figure 6 has a resistance of 60 ohms, the current drawn by each lamp is 2 amps. The current in line 1 is 4 amps, and the current in line 2 is also 4 amps. The current in the neutral conductor is zero.

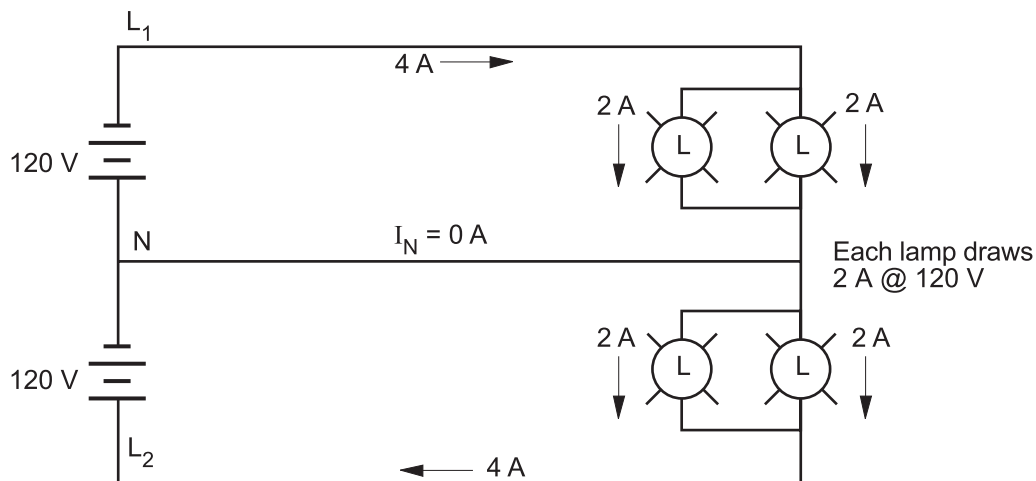


Figure 6—Balanced three-wire circuit with 120 V loads

A balanced three-wire circuit exists when each of the line conductors carries equal currents, and the neutral current is zero. Although the preceding circuit shows only 120 V loads, a 240 V load may also be supplied and balanced, as shown in Figure 7.

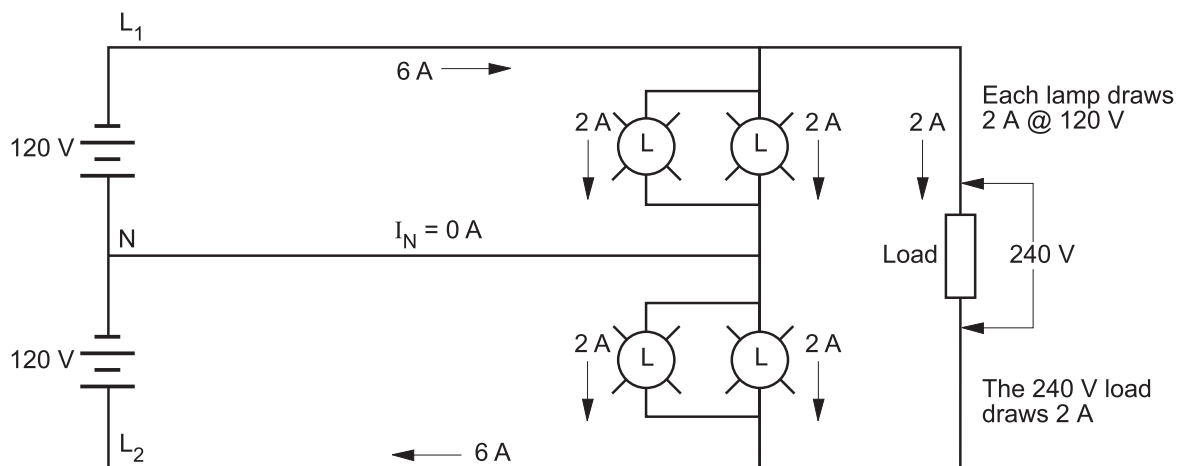


Figure 7—Balanced three-wire circuit with 120 V and 240 V loads

If a 240 V load drawing 2 amps is connected to the circuit, the only difference is that the current in each line conductor is increased by 2 amps. Lines 1 and 2 now carry 6 amps each and the neutral current is still zero. This is still a balanced three-wire circuit.



Note the directions of current flow. Apply Kirchhoff's current law: The amount of current entering a junction must equal the amount of current leaving that junction.

Since there is current flow in only two of the three system conductors, balanced loading reduces line loss. Ideally, we like to balance circuit loading, but it is not always possible to do so.

Unbalanced three-wire circuits

If an unequal amount of load resistance is connected between each of the 120 V line-to-neutral sections of a three-wire circuit, we have an unbalanced load condition. In Figure 8, for example, three 120 V lamps (each drawing 2 amps) are connected to the circuit. You can see that line 1 delivers 2 amps to its lamp, but the other bank of lamps requires 4 amps. To accomplish this, the neutral conductor must now deliver 2 amps (by Kirchhoff's current law) to provide 4 amps to the remaining lamps. The current in line 2 is now 4 amps.

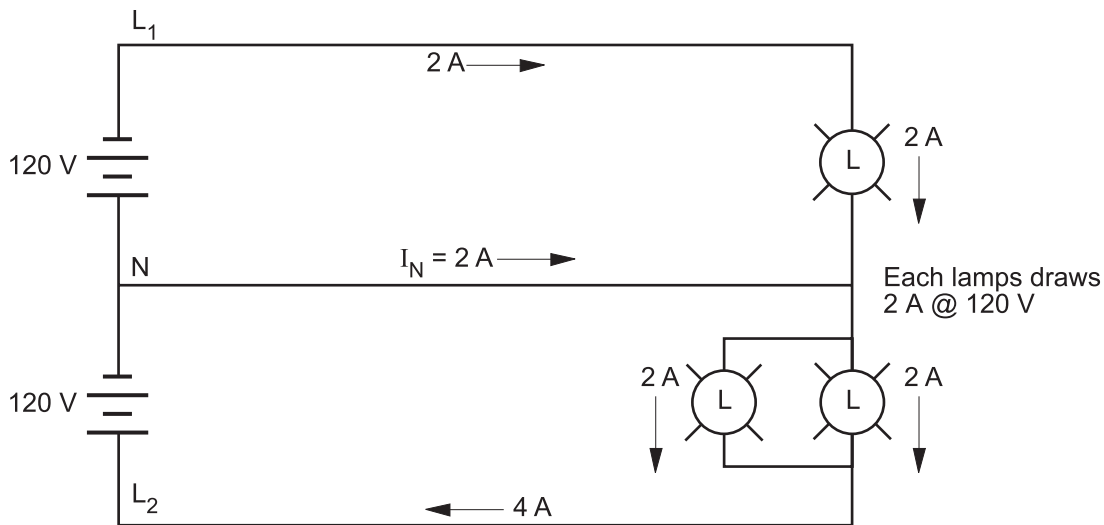


Figure 8—Unbalanced three-wire circuit with 120 V loads



Note the directions of current flow, and apply Kirchhoff's current law at each of the junctions!

In an unbalanced three-wire circuit, the line currents are not equal, and the neutral conductor carries current. Notice that the neutral always carries the difference between the two line currents (in the example in Figure 8, $4\text{ A} - 2\text{ A} = 2\text{ A}$).

Again, if a 240 V load drawing 2 amps is also connected, the only difference is an increase in the two line currents. The neutral current is still the same. In Figure 9, apply Kirchhoff's current law to each of the junctions in the circuit—observing the direction of current flow—and confirm the values shown.

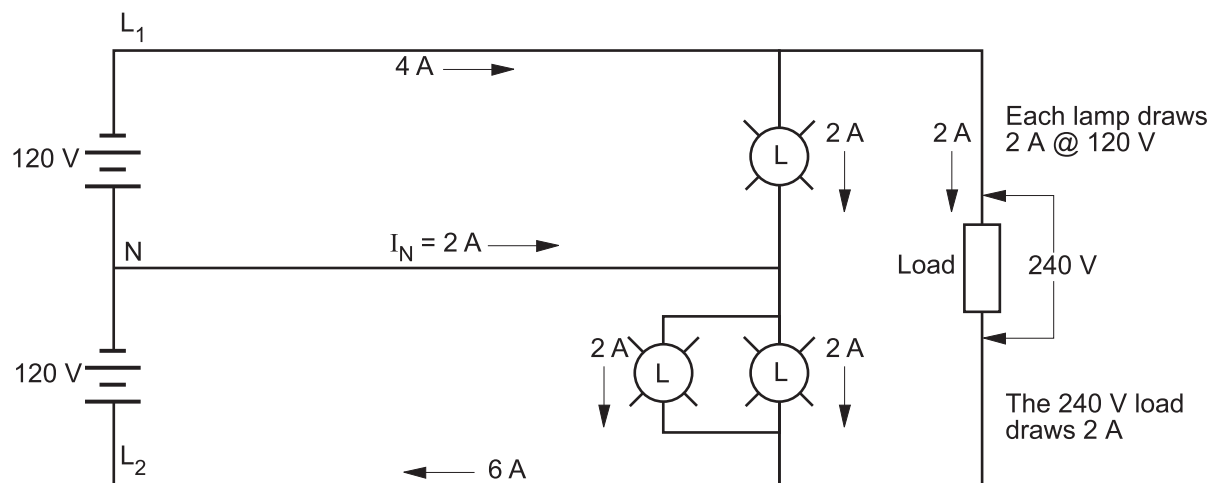


Figure 9—Unbalanced three-wire circuit with 120 V and 240 V loads

Importance of the neutral

Up to this point, you have observed that if the loads on a three-wire circuit are balanced, the neutral conductor carries no current. And, if the three-wire circuit is unbalanced, the neutral conductor carries the difference between the two line currents, or the unbalanced portion of the circuit load.

Whenever possible, distribute loads on a three-wire circuit so that a balanced situation occurs and there is zero current in the neutral. One reason for this is that the neutral maintains the voltage across line-to-neutral loads at 120 V for both balanced and unbalanced loads.

If the neutral conductor ever develops a break (or becomes open), certain problems may be encountered. First, consider a balanced three-wire circuit, as shown previously in Figure 6. Since both line-to-neutral load-banks draw the same current, the neutral current is zero. Now, consider what happens if the neutral develops a break, as shown in Figure 10.

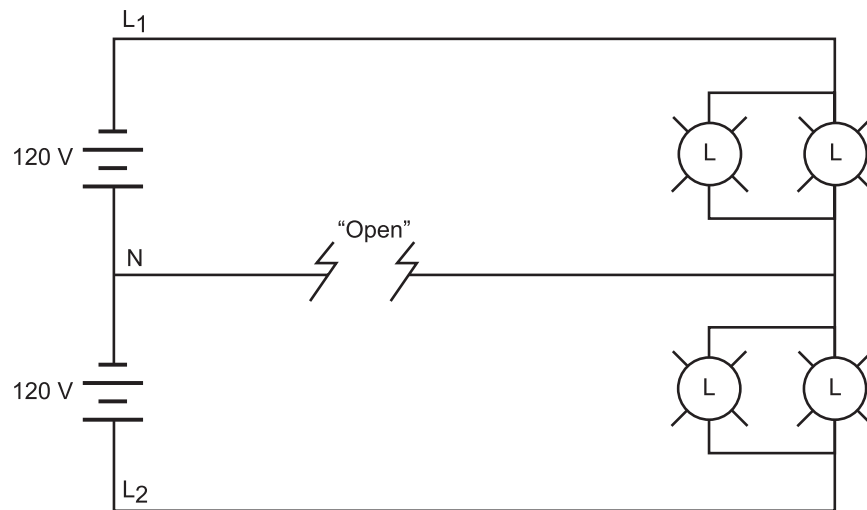


Figure 10—Balanced loads with an open neutral

If the neutral becomes open, then the two banks of lamps are connected in series across 240 V. Based on the previous condition, the resistance of each bank is calculated as 30 ohms ($120\text{ V} \div 4\text{ A}$). We can simplify this new circuit condition as shown in Figure 11.

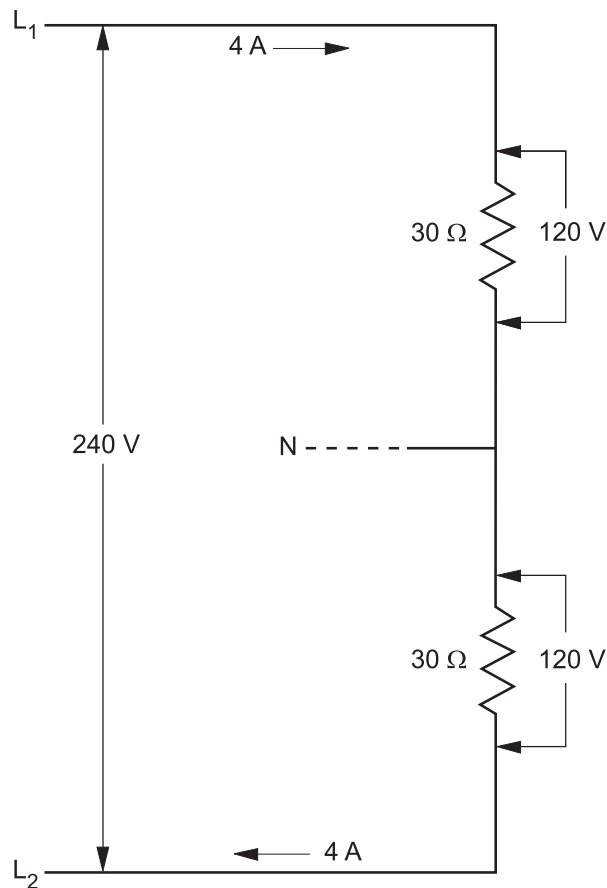


Figure 11—Simplified balanced circuit with open neutral

The current drawn from the 240 V supply is calculated as $240\text{ V} \div 60\ \Omega = 4\text{ A}$, which is the same as before. The voltage drop across each load bank is $4\text{ A} \times 30\ \Omega = 120\text{ V}$ —also the same as before.

Therefore, if the three-wire circuit is balanced, a break (open) in the neutral does not change any values in the circuit. But what if the circuit is unbalanced? As shown before in Figure 8, different currents occur as a result of the unbalanced loads connected in the circuit.

- The 2 A load has a resistance of $120\text{ V} \div 2\text{ A} = 60\ \Omega$.
- The 4 A load has a resistance of $120\text{ V} \div 4\text{ A} = 30\ \Omega$.

As with a balanced load, if the neutral becomes open, the two loads are connected in series across 240 V. This, however, has different results. Let's redraw the circuit in a simplified form as in Figure 12.

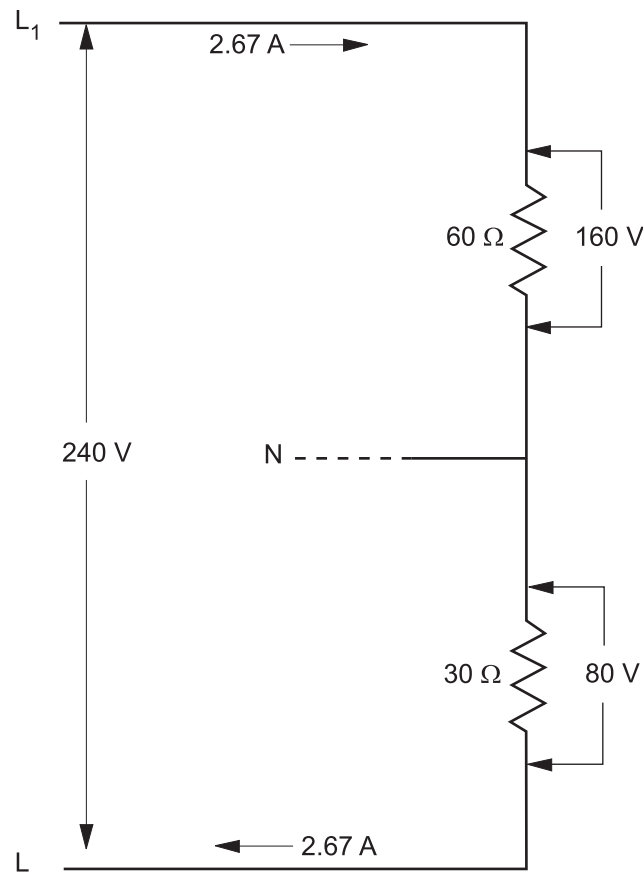


Figure 12—Simplified unbalanced circuit with open neutral

Note that there are unequal resistances connected in series across 240 V.

- Total circuit current is calculated at $240\text{ V} \div 90\ \Omega = 2.67\text{ A}$.
- The voltage drop across the 60 ohm load is $2.67\text{ A} \times 60\ \Omega = 160\text{ V}$.
- The voltage drop across the 30 ohm load is $2.67\text{ A} \times 30\ \Omega = 80\text{ V}$.

If all of the lamps are rated at 120 V, the 80 V drop will not hurt the lamps—they will just glow considerably dimmer. However, if 160 V is impressed across a lamp rated at 120 V, it will glow considerably brighter and burn out very quickly.

If the circuit was even more unbalanced than illustrated, the resulting voltage imbalance would be even more severe—causing serious damage to equipment.

The preceding demonstrates the importance of the neutral. Under unbalanced loading, it maintains equal voltages across the loads by conducting the current imbalance back to the appropriate 120 V source.



Never open the neutral or fuse the neutral conductor. Keep it continuous as much as possible, with as few splices as possible. A loose or poor electrical connection in the neutral conductor results in increased resistance, which in turn alters the load voltages.



Now do Self-Test 13 and check your answers.

Self-Test 13

1. In a single-phase, three-wire system:

a. What are the three conductors called?

b. How are they colour-coded?

2. In a three-wire system, why do we attempt to balance the loads connected to each line conductor?

3. If the neutral wire is broken or disconnected from a three-wire circuit, what is the danger to equipment?

4. What is the neutral current in a three-wire system when the two line currents are 12 A and 15 A?

5. Why is it not permitted to fuse the neutral in a three-wire circuit?

6. In a 120 V/240 V, three-wire circuit:

- An $8\ \Omega$ resistor is connected between line 1 and neutral.
- A $12\ \Omega$ resistor is connected between line 2 and neutral.
- A $24\ \Omega$ resistor is connected between line 1 and line 2.

Determine:

a. the total current in line 1

b. the total current in line 2

c. the current in the neutral

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 14:

Solve problems involving three-wire circuits

You have been shown simple calculations for three-wire circuits, and you have applied Kirchhoff's current law to show the directions of current flow in line and neutral conductors. In preceding calculations, line resistance was neglected. To obtain more accurate results, and to show the unique effects of line voltage drops, the resistance of the conductors will now be considered.

Voltage and current calculations

To determine load voltages, we must apply Kirchhoff's voltage law, which states that:

The sum of the voltage drops or rises in any closed circuit must equal the source voltage in that circuit.

First, let's review a simple two-wire circuit, taking conductor resistance into account. In Figure 1, a load draws 2 amps from a 120 V source. The resistance of the conductors is as shown.

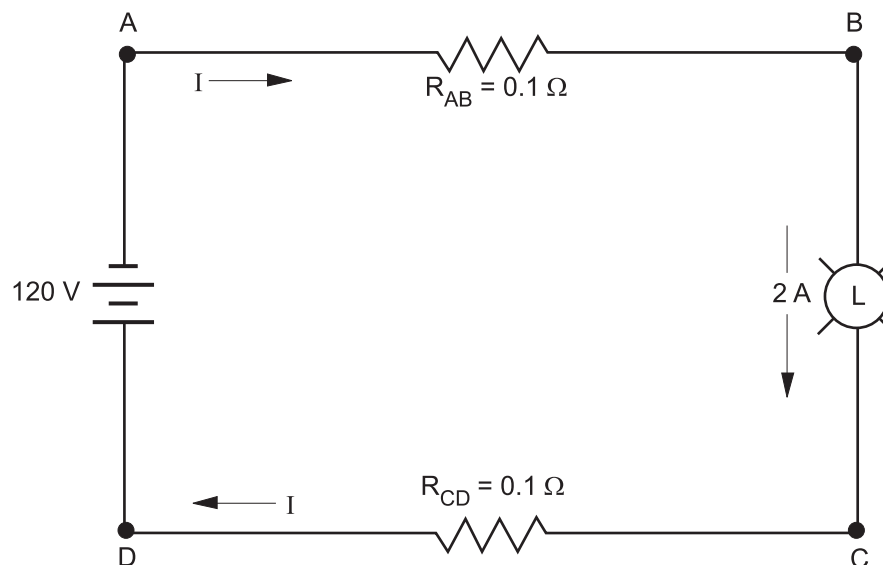


Figure 1—Simple two-wire circuit

An important step in any circuit analysis is to determine the direction of current flow through the closed loops. Whenever you move through a loop in the same direction, each voltage is considered a drop. The source voltage must equal the sum of all the voltage drops.

Apply Kirchhoff's voltage law to the simple circuit shown in Figure 1. The loop equation is:

$$V_{\text{SUPPLY}} = V_{AB} + V_{BC} + V_{CD}$$

Using Ohm's law and substituting, the voltage across the load gives the following:

$$120 \text{ V} = (2 \text{ A} \times 0.1 \Omega) + V_{BC} + (2 \text{ A} \times 0.1 \Omega)$$

$$120 \text{ V} = 0.2 \text{ V} + V_{BC} + 0.2 \text{ V}$$

$$V_{BC} = 120 \text{ V} - 0.4 \text{ V}$$

$$= 119.6 \text{ V}$$

Now let's analyze a simple three-wire circuit that includes conductor resistance. For the values of load current shown, you can show the direction (as illustrated in Figure 2) and determine the values of each of the currents:

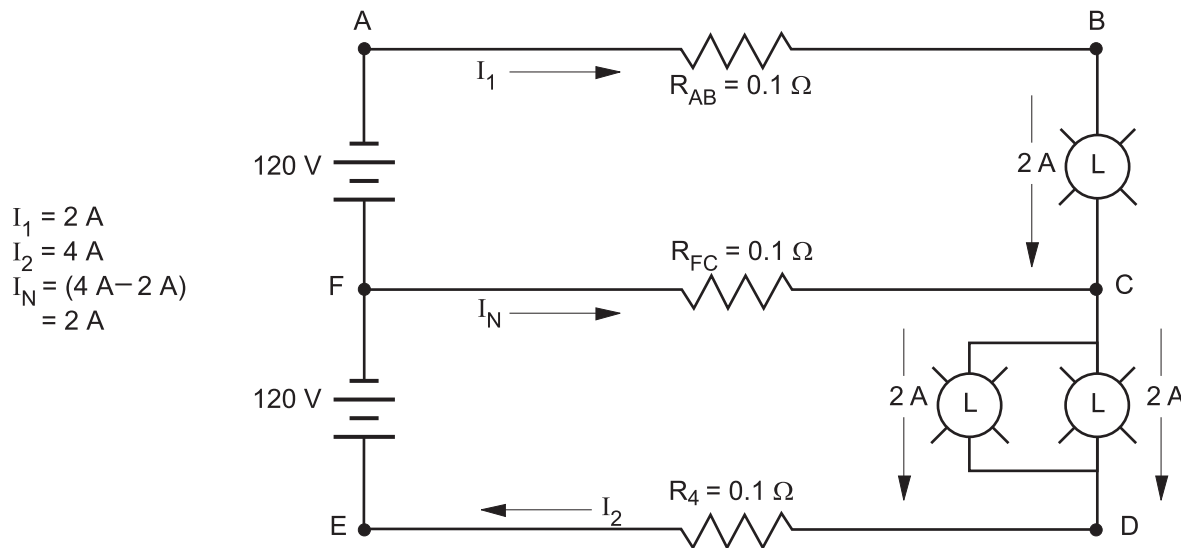


Figure 2—Simple unbalanced three-wire circuit

Remember, whenever you move through a closed loop in the same direction from the source, each voltage is considered a voltage drop and the supply voltage must equal the sum of all the voltage drops. But when current flows in an opposite direction in tracing a closed loop, it results in a voltage *rise*. Whenever we encounter a voltage rise, it is not added to all the other voltage drops, but subtracted from them, so that the total voltage drop will equal the supply voltage.

Use Figure 2 as an example. When you trace through the upper closed loop from the supply V_{AF} , you must move *against* the direction of current I_N in the part of the loop marked $C \rightarrow F$.

To obtain the load voltage V_{BC} in the upper loop, look at this loop as follows:

$$V_{\text{SUPPLY}} = V_{AB} + V_{BC} - V_{CF}$$

$$120 \text{ V} = (2 \text{ A} \times 0.1 \Omega) + V_{BC} - (2 \text{ A} \times 0.1 \Omega)$$

$$120 \text{ V} = 0.2 \text{ V} + V_{BC} - 0.2 \text{ V}$$

$$V_{BC} = 120 \text{ V}$$

Now consider the lower closed loop in Figure 2. In tracing this circuit, you find that you travel in the same direction as all the currents. Therefore, the voltage V_{CD} across the load in the lower loop is found by:

$$V_{\text{SUPPLY}} = V_{FC} + V_{CD} + V_{DE}$$

$$120 \text{ V} = (2 \text{ A} \times 0.1 \Omega) + V_{CD} + (4 \text{ A} \times 0.1 \Omega)$$

$$120 \text{ V} = 0.2 \text{ V} + V_{CD} + 0.4 \text{ V}$$

$$V_{CD} = 119.4 \text{ V}$$

You can confirm the calculated load voltages in each part of the three-wire circuit by considering the outer loop across the total supply voltage of 240 V. Again by tracing the loop, you can establish that:

$$V_{\text{SUPPLY}} = V_{AB} + V_{BC} + V_{CD} + V_{DE}$$

$$= (2 \text{ A} \times 0.1 \Omega) + 120 \text{ V} + 119.4 \text{ V} + (4 \text{ A} \times 0.1 \Omega)$$

$$= 0.2 \text{ V} + 120 \text{ V} + 119.4 \text{ V} + 0.4 \text{ V}$$

$$= 240 \text{ V}$$

You can see that the source voltage equals the sum of all the voltage drops, which confirms Kirchhoff's voltage law.

Now consider a more complicated three-wire circuit and determine load voltages, using the voltage drop versus the voltage rise concept. In the three-wire circuit of Figure 3, you are given the current in each load and the resistance of the wires. You are required to calculate the voltage across each load.

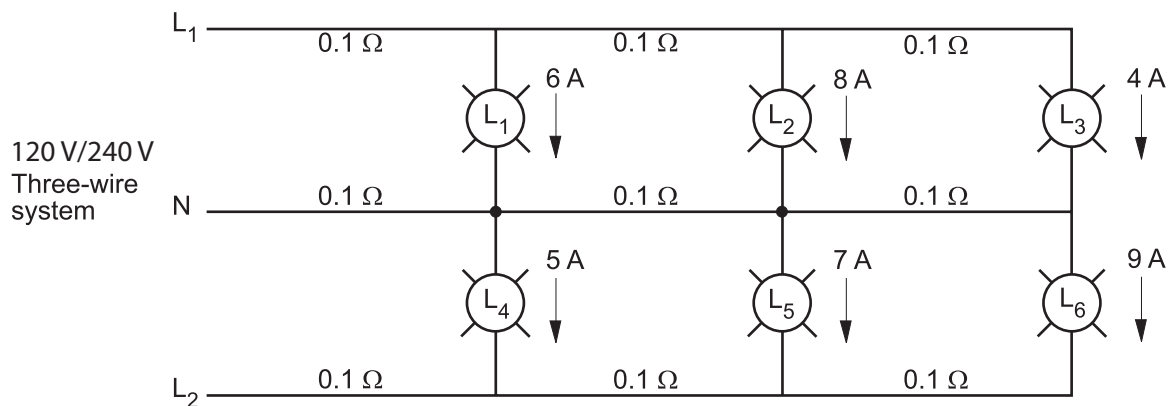
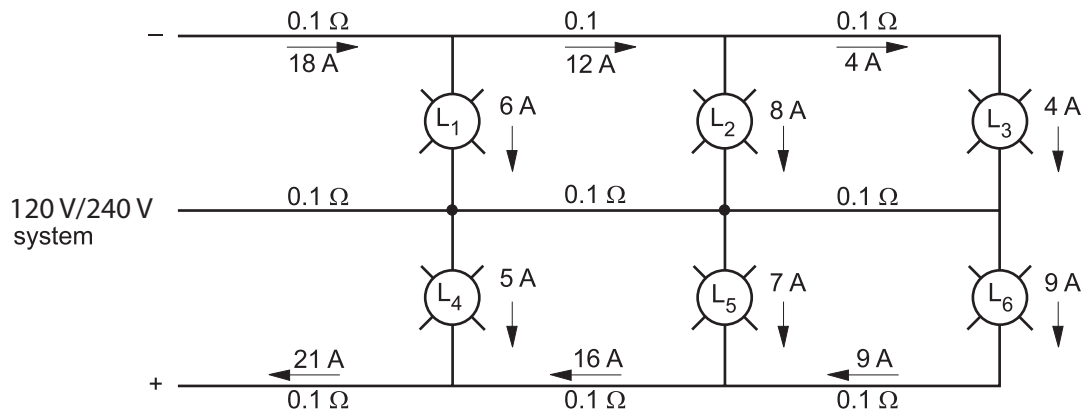


Figure 3—Unbalanced three-wire circuit

Following on the next page is an outline of the step-by-step procedure for the required calculations.

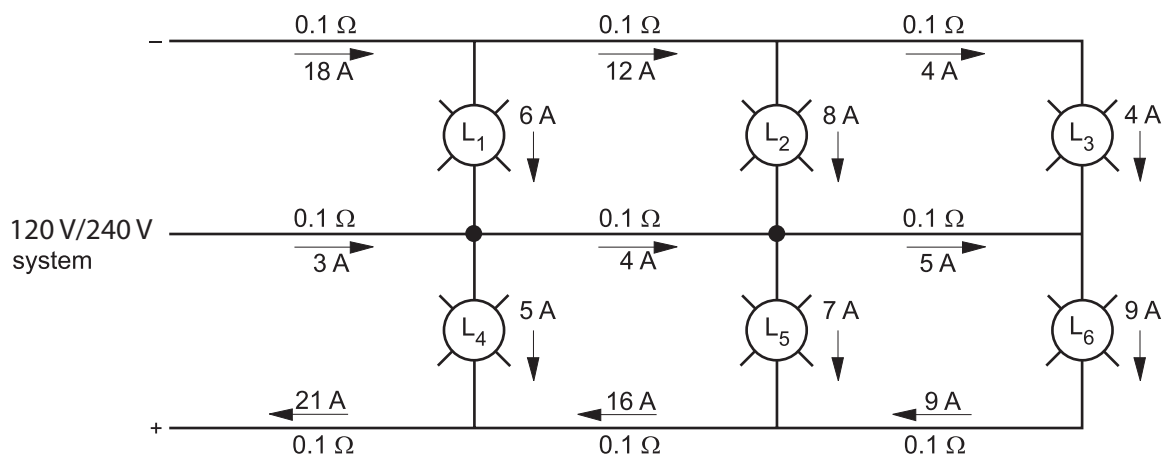
Step 1

- Assume a direction of current flow in the lines and mark these directions in the different sections of the hot wires, as shown in Figure 4.
- Starting at the far right, apply Kirchhoff's current law at each junction, as you work back to the supply, and mark in the current values.

**Figure 4—Current directions in line conductors****Step 2**

The current in the neutral wire in any section is always the difference between the currents in the hot wires in that section. Its direction is the same as that of the smaller of the two hot wire currents.

Mark the currents in the neutral and their respective direction, as shown in Figure 5. The values and directions found in Steps 1 and 2 are important in determining the voltage drops or voltage rises in the circuit.

**Figure 5—Current directions in neutral**

Step 3

The voltage drop in any conductor section is equal to the current flowing in that section multiplied by the wire resistance of that section. Mark the voltage drops in the wires sections as shown in the boxes in Figure 6.

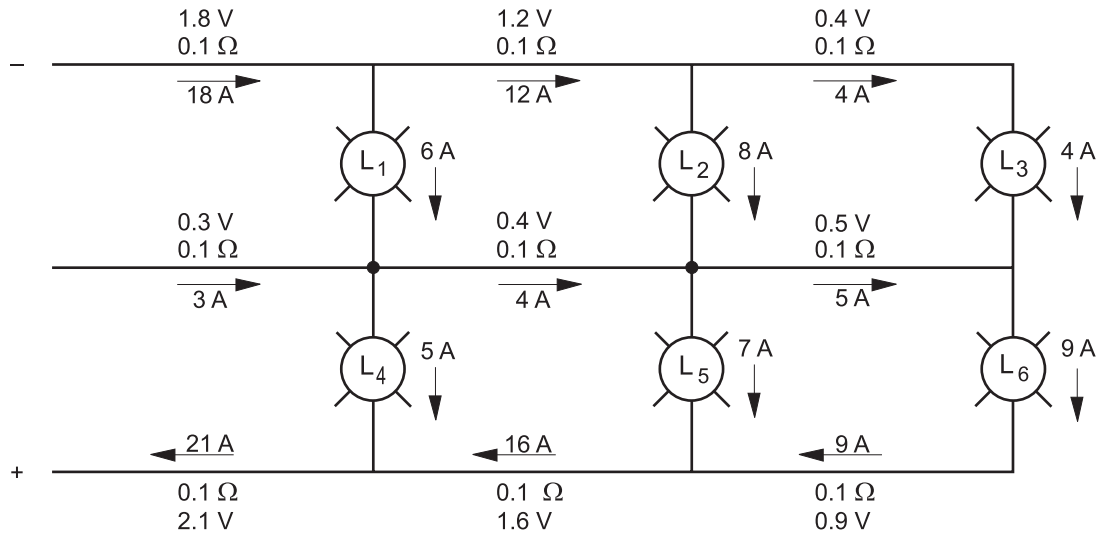


Figure 6—Line-drop values

Step 4

Using the closed loop, as shown by the broken line in Figure 7, apply Kirchhoff's voltage law to solve the voltage of load L_1 . It is usually easiest to use a *loop equation* to find the voltage at any of the loads. The procedure is as follows:

- Assume a clockwise direction around the loop.
- Where the equation follows the current direction, the emfs are voltage drops and are subtracted from the reference (or supply) voltage.
- Where the neutral current is counter to the equation, the emfs are voltage rises and are added to the reference voltage.

First, considering loop 1 in Figure 7, the equation is:

$$\begin{aligned} V_{L1} &= 120 \text{ V} - 1.8 \text{ V} + 0.3 \text{ V} \\ &= 118.5 \text{ V} \end{aligned}$$

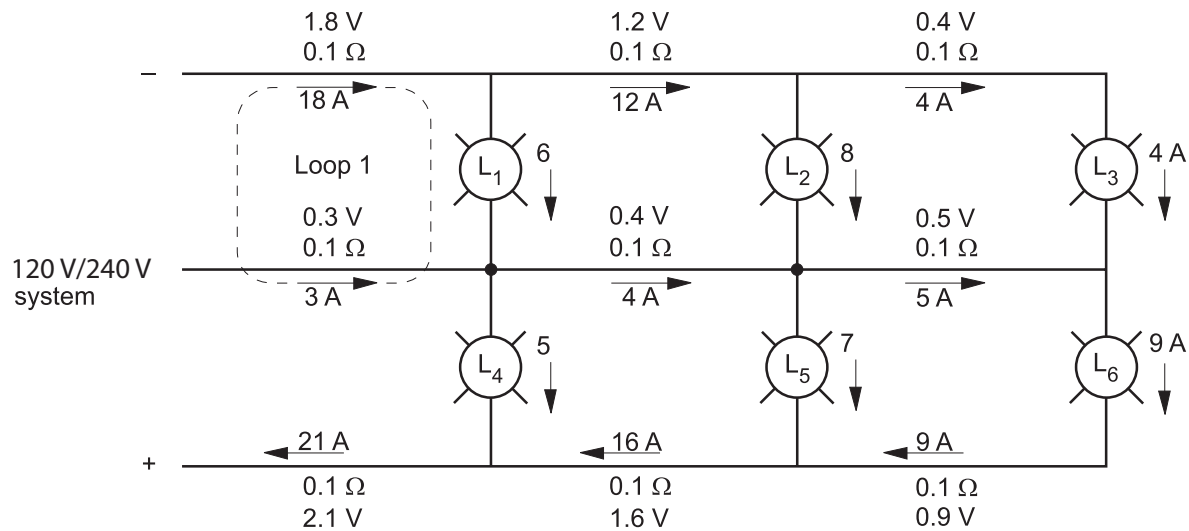


Figure 7—Voltage calculations

Step 5

Repeat the procedure of Step 4 for each of the remaining loops, to solve for the load voltage. The reference (or source) voltage for each loop can be taken from the previous equation. For example, the reference voltage for loop 2 is $V_{L1} = 118.5 \text{ V}$; the reference voltage for loop 3 is V_{L2} and so on.

Refer to Figure 8 for the remaining loop equations.

For loop 2:

$$\begin{aligned} V_{L2} &= 118.5 \text{ V} - 1.2 \text{ V} + 0.4 \text{ V} \\ &= 117.7 \text{ V} \end{aligned}$$

For loop 3:

$$\begin{aligned} V_{L3} &= 117.7 \text{ V} - 0.4 \text{ V} + 0.5 \text{ V} \\ &= 117.8 \text{ V} \end{aligned}$$

For loop 4:

$$\begin{aligned} V_{L4} &= 120 \text{ V} - 0.3 \text{ V} - 2.1 \text{ V} \\ &= 117.6 \text{ V} \end{aligned}$$

For loop 5:

$$\begin{aligned} V_{L5} &= 117.6 \text{ V} - 0.4 \text{ V} - 1.6 \text{ V} \\ &= 115.6 \text{ V} \end{aligned}$$

For loop 6:

$$\begin{aligned} V_{L6} &= 115.6 \text{ V} - 0.5 \text{ V} - 0.9 \text{ V} \\ &= 114.2 \text{ V} \end{aligned}$$

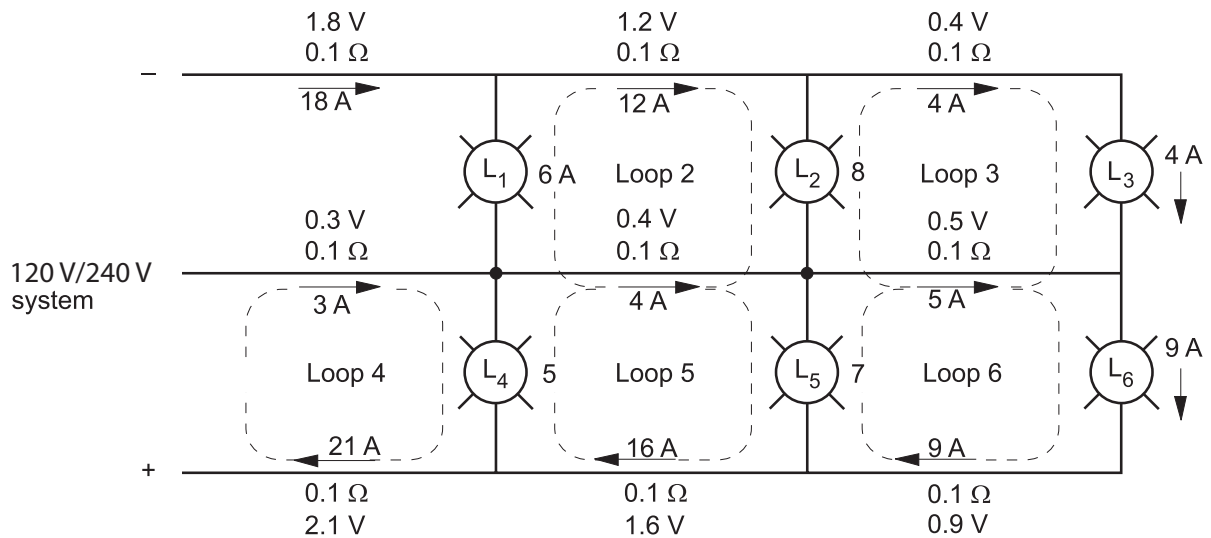


Figure 8—Loop analysis

By analyzing each of the loops, and by differentiating between voltage drops and voltage rises, you can solve for the voltages across each load. By following logical procedural steps, a relatively complicated three-wire circuit can be solved quite easily.

Refer to each of the preceding diagrams, and review each step until you feel confident about the reasoning described.

Open neutral calculations

You have already seen the importance of the neutral in a three-wire circuit. Keep in mind these facts while calculating open neutral circuits:

- In a balanced three-wire circuit, you would expect the neutral to carry zero current.
- However, if the three-wire circuit is unbalanced, the neutral conducts the difference between the two line currents to maintain balanced voltages across the loads.
- If the connected loads are severely unbalanced *and* the neutral is opened, the voltages across the loads also become severely unbalanced. The reason for this is that the system reverts to a series circuit across 240 V, and unequal resistors in series have unequal voltages across them.
- Therefore, never:
 - fuse the neutral
 - switch the neutral
 - and never open the neutral while working on a live system!

Load voltages in an unbalanced three-wire circuit

Consider the severely unbalanced three-wire circuit in Figure 9. (The line conductor resistance is included for these calculations.)

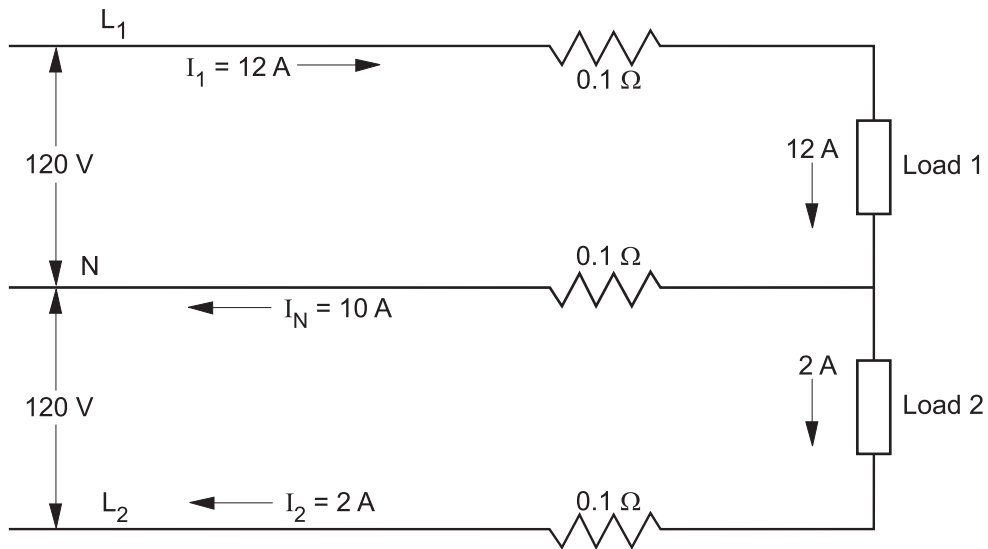


Figure 9—Severely unbalanced three-wire circuit

The voltage across the 12 A load is calculated as:

$$\begin{aligned} E_1 &= 120 \text{ V} - (12 \text{ A} \times 0.1 \Omega) - (10 \text{ A} \times 0.1 \Omega) \\ &= 120 \text{ V} - 1.2 \text{ V} - 1 \text{ V} \\ &= 117.8 \text{ V} \end{aligned}$$

The voltage across the 2 A load is calculated as:

$$\begin{aligned} E_2 &= 120 \text{ V} + (10 \text{ A} \times 0.1 \Omega) - (2 \text{ A} \times 0.1 \Omega) \\ &= 120 \text{ V} + 1 \text{ V} - 0.2 \text{ V} \\ &= 120.8 \text{ V} \end{aligned}$$

You can see that the two load voltages are only slightly unbalanced, even though the load currents are severely unbalanced. The neutral connection helps to stabilize the voltages somewhat.

Effect of an open neutral on load voltages

Now let's observe the effect of an open neutral, as illustrated in Figure 10.

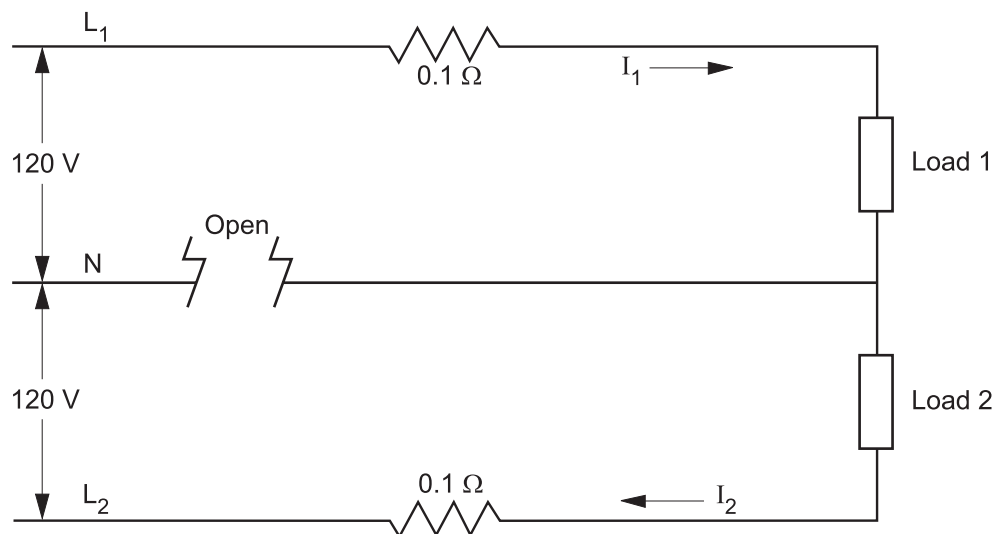


Figure 10—Open neutral three-wire circuit

- Since the neutral is open, there can be no neutral current.
- The two loads are connected in series across the line-to-line voltage of 240 V.
- Recognize that the line currents are NOT the same as before!

To determine the new line currents, you must first solve for the connected resistance of each load. This is based on the previous current demands and voltages.

The resistance of load 1 is:

$$R_1 = 117.8 \text{ V} \div 12 \text{ A} = 9.82 \Omega$$

The resistance of load 2 is:

$$R_2 = 120.8 \text{ V} \div 2 \text{ A} = 60.4 \Omega$$

You could further simplify this circuit as shown in Figure 11.

Notice that since we have a series circuit, the two line currents are the same. The line current may be calculated as follows:

$$I_1 = I_2 = 240 \text{ V} \div 70.42 \Omega = 3.41 \text{ A}$$

The total line (voltage) drop in the conductors is:

$$E = 3.41 \text{ A} \times 0.2 \Omega = 0.68 \text{ V}$$

The voltage across each of the loads is:

$$E_{L1} = 3.41 \text{ A} \times 9.82 \Omega = 33.49 \text{ V}$$

$$E_{L2} = 3.41 \text{ A} \times 60.41 \Omega = 205.96 \text{ V}$$

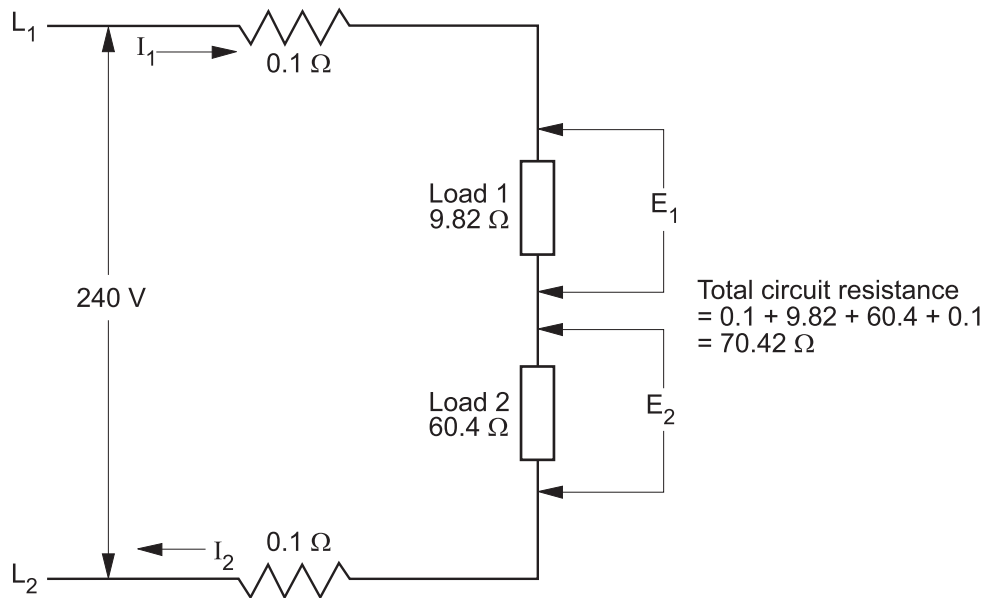


Figure 11—Simplified circuit

You can now see how greatly unbalanced the two load voltages are. Load 2 is subjected to an abnormally high voltage and could burn out. Load 1 has a considerably lower than normal voltage and probably won't operate.

Voltage across the break of an open neutral circuit

One final analysis of a three-wire circuit remains, and that is how to calculate the voltage across the break of an open neutral circuit. Consider the simple circuit shown in Figure 12. Line resistance will be ignored for this calculation, and DC sources are used again so that instantaneous polarities can be marked.

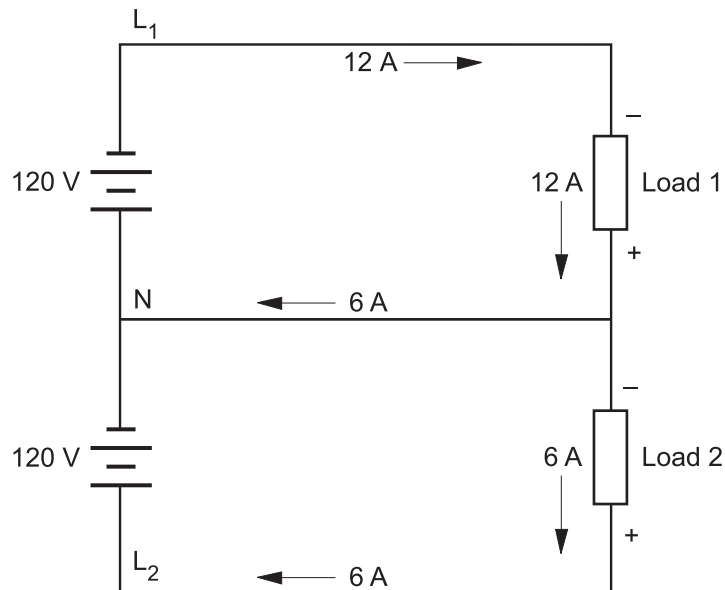


Figure 12—Unbalanced three-wire circuit

Based on the values shown in Figure 12, the resistance of each load can be easily determined.

$$\text{Load 1} = 120 \text{ V} \div 12 \text{ A} = 10 \, \Omega$$

$$\text{Load 2} = 120 \text{ V} \div 6 \text{ A} = 20 \, \Omega$$

If the neutral becomes open as shown in Figure 13, the two loads are in series across 240 V. The line current is then calculated as:

$$I_1 = 240 \text{ V} \div 30 \, \Omega = 8 \text{ A}$$

And now using the line current, the voltage across each load is calculated as follows:

$$E_1 = 8 \text{ A} \times 10 \, \Omega = 80 \text{ V}$$

$$E_2 = 8 \text{ A} \times 20 \, \Omega = 160 \text{ V}$$



Note the polarity of the voltage across each load with respect to the 120 V supply polarities.

If a voltmeter is connected across the break shown in Figure 13, what does it indicate?

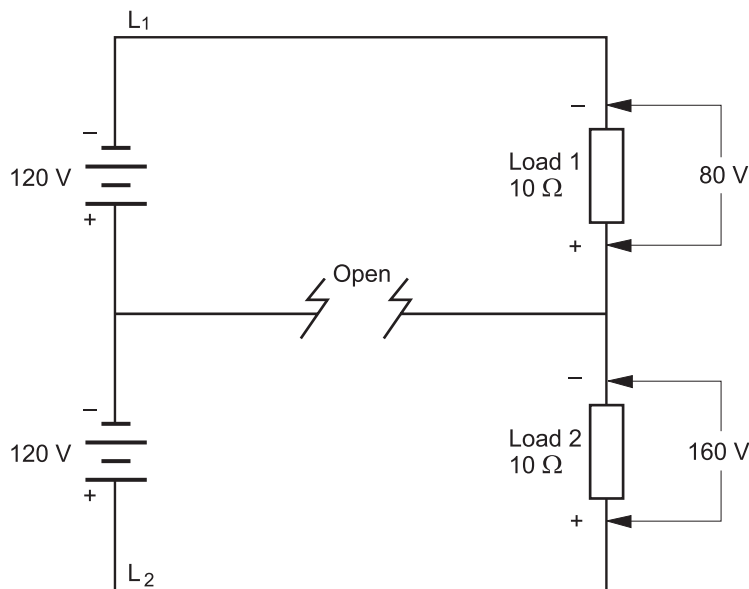


Figure 13—Open neutral condition

To illustrate this condition, only the upper portion of the preceding three-wire circuit is shown in Figure 14.

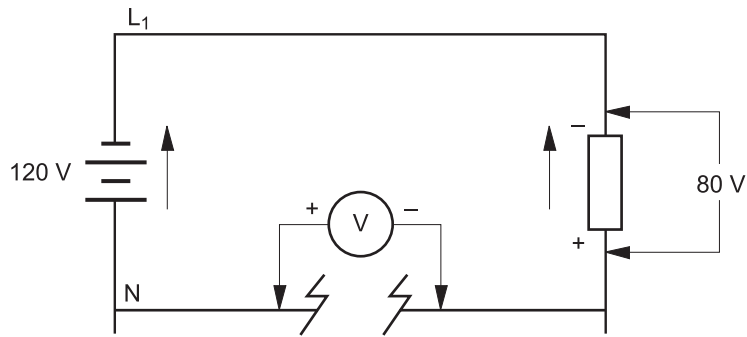


Figure 14—Voltmeter across the break (open)

Kirchhoff's voltage law states that all the voltage drops in a circuit must equal all of the voltage rises. If the source voltage is 120 V, and if there is a voltage drop of 80 V across load 1, then the voltage across the break must be 40 V. See Figure 15.

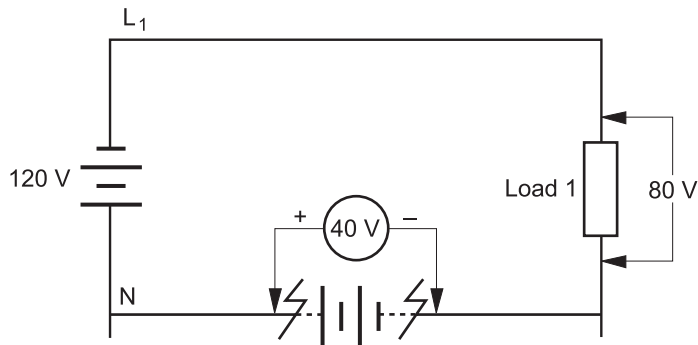


Figure 15—Voltage drop across the open

Looking at only the lower portion of this three-wire circuit, you see a similar situation. Refer to Figure 16.

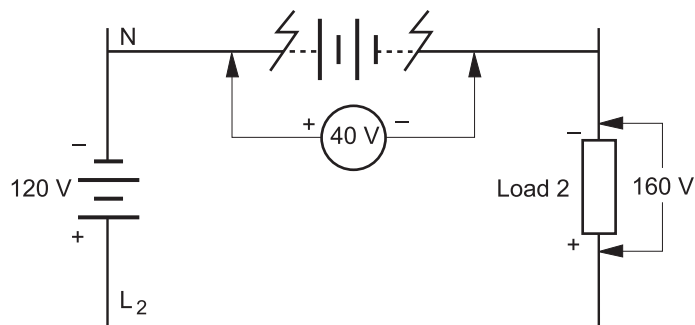


Figure 16—Voltage rise across the open

With respect to load 2, the voltage across the open acts as a voltage rise to give a load voltage of 160 V. Notice that the polarity and value of the voltage across the open are the same in each half of the load circuit.



Note the similarities in Figures 15 and 16.

Voltage across the break in a balanced three-wire circuit with open neutral

If you analyze a balanced three-wire circuit with an open neutral, you notice that the voltage across the break is zero.

Convert Figure 12 to a balanced circuit and apply the procedure illustrated above to confirm this.



Now do Self-Test 14 and check your answers.

Self-Test 14

Each of the three-wire circuits in Figures 1, 2 and 3 has line resistance values and currents as indicated. In each case, solve for the voltages V_{L1} , V_{L2} , V_{L3} and V_{L4} expected across the loads.

1.

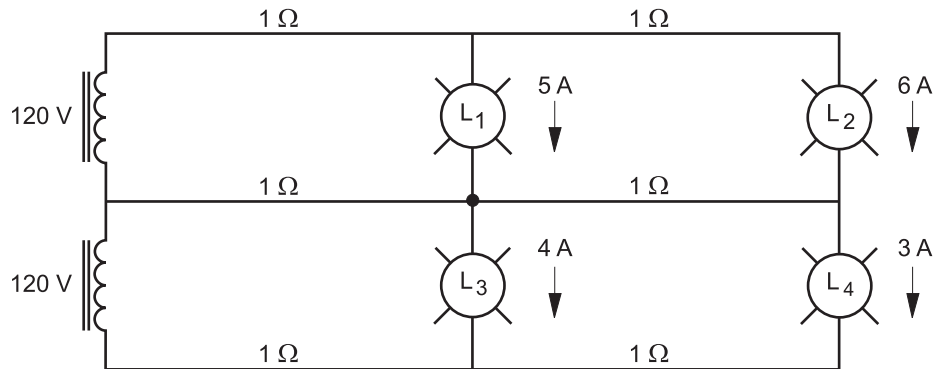


Figure 1—Circuit for Question 1

_____	_____
_____	_____

2.

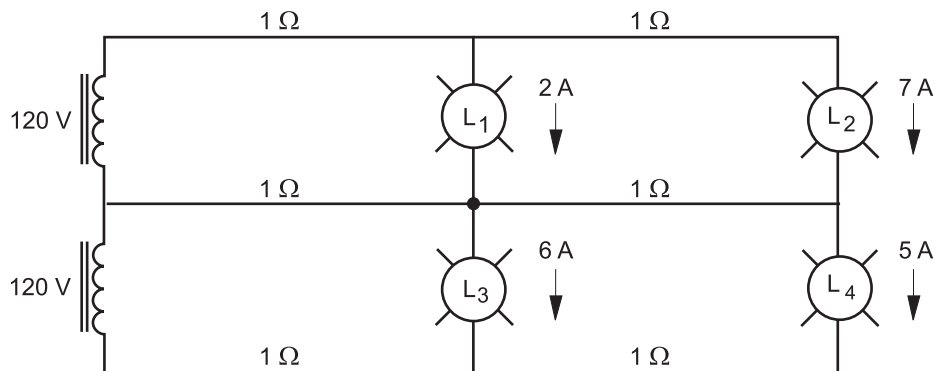


Figure 2—Circuit for Question 2

_____	_____
_____	_____

3.

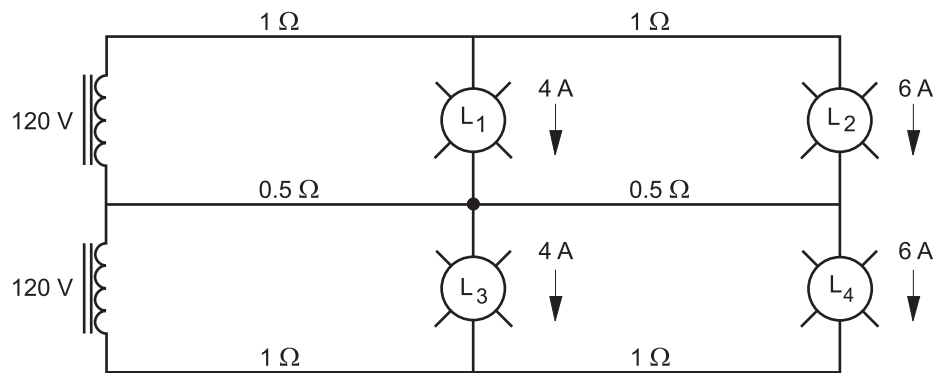


Figure 3—Circuit for Question 3

For each three-wire circuit in Figures 4, 5 and 6, current values under normal operating conditions are shown.

4. If the neutral developed an open at point x, calculate the following:

a. voltage across load 1

b. voltage across load 2

c. total line current

d. total line (voltage) drop

e. voltage across the open

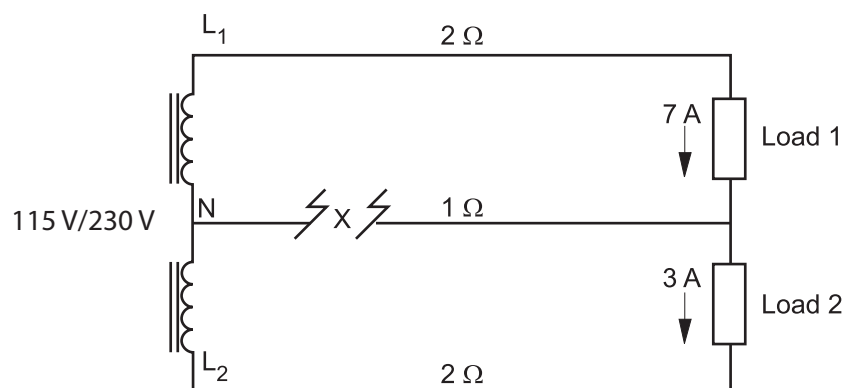


Figure 4—Circuit for Question 4

5. If the three-wire circuit developed an open as shown in Figure 5, calculate the following:

- a. voltage across load 1 _____
 - b. voltage across load 2 _____
 - c. voltage across load 3 _____
 - d. voltage across load 4 _____
 - e. total line current _____
 - f. voltage across the open _____
- (Neglect line drops in this calculation.)

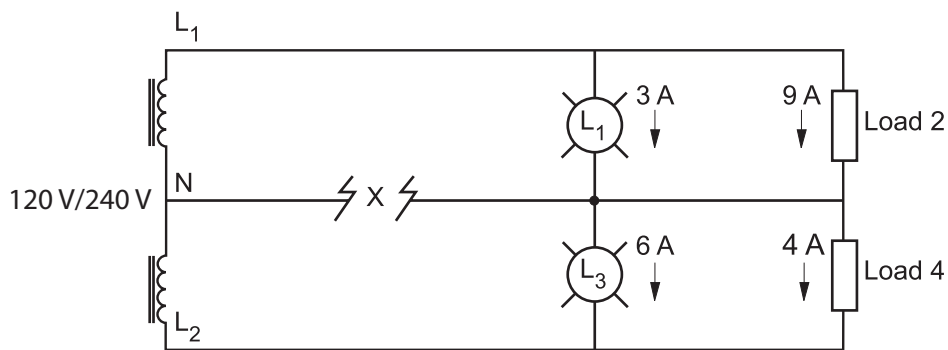


Figure 5—Circuit for Question 5

6. In Figure 6, if the neutral became open at point X, calculate the following:

- a. voltage across load 1 _____
 - b. voltage across load 2 _____
 - c. voltage across load 3 _____
 - d. voltage across load 4 _____
 - e. total line current _____
 - f. voltage across the open _____
- (Neglect line drops in this calculation.)

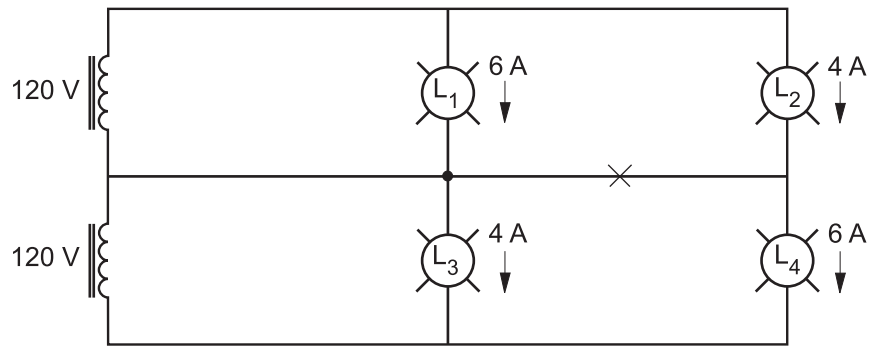


Figure 6—Circuit for Question 6

Go to the Answer Key at the end of the Learning Guide to check your answers.

Answer Key

Self-Test 1

1. one
2. a. resistance
3. c. more than before
4. Neither. Both ammeters are marked with incorrect polarity.
5. b. less than the voltage drop across the $8\ \Omega$ resistor
6. $24\ \Omega$
7. $6\ \Omega$
8. $12\ \text{V}$
9. $60\ \text{V}$
10. d. equals the applied voltage
11. a. There is only one path for current through both resistors.
12. b. An open in the circuit interrupts current to all the devices.
13. d. $120\ \text{V}$

Self-Test 2

1. $8.39\ \text{A}$
2. $5\ \Omega$
3. $0.082\ \text{A}$; $E_1 = 20.5\ \text{V}$; $E_2 = 38.6\ \text{V}$; $E_3 = 41\ \text{V}$
4.
 - a. $E_1 = 20\ \text{V}$
 - b. $E_2 = 60\ \text{V}$
 - c. $E_3 = 40\ \text{V}$
 - d. $R_3 = 50\ \Omega$
 - e. $R_T = 150\ \Omega$
 - f. $P_T = 96\ \text{W}$

5.
 - a. $E_1 = 24 \text{ V}$
 - b. $E_2 = 16 \text{ V}$
 - c. $E_3 = 20 \text{ V}$
 - d. $E_T = 60 \text{ V}$
 - e. $R_T = 15 \Omega$
 - f. $P_2 = 64 \text{ W}$
 - g. $P_3 = 80 \text{ W}$
 - h. $P_T = 240 \text{ W}$
6.
 - a. $E_1 = 36 \text{ V}$
 - b. $E_2 = 54 \text{ V}$
 - c. $R_2 = 9 \Omega$
 - d. $R_3 = 5 \Omega$
 - e. $R_T = 20 \Omega$
 - f. $P_T = 720 \text{ W}$
7.
 - a. $E_1 = 50 \text{ V}$
 - b. $E_2 = 45 \text{ V}$
 - c. $E_T = 120 \text{ V}$
 - d. $R_3 = 50 \Omega$
 - e. $R_1 = 100 \Omega$
 - f. $P_T = 60 \text{ W}$
8. 2.5 A
9. 0.714 A
10. 150Ω
11. rheostat
12. c. watts
13. b. volts
14. b. decreases
15. a. increases

16. a. 4 V
b. 20 W

Self-Test 3

1. Polarities and voltages for Self-Test 3 are shown in Figures 1, 2 and 3.

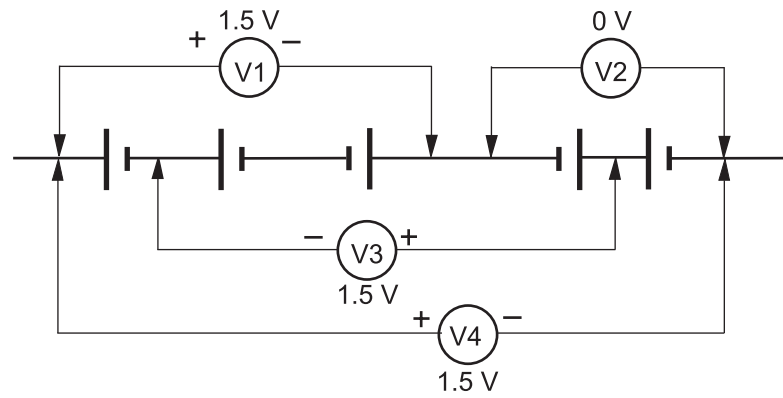


Figure 1

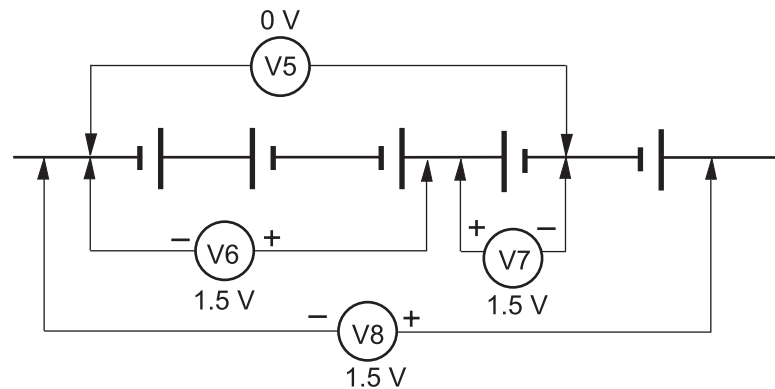


Figure 2

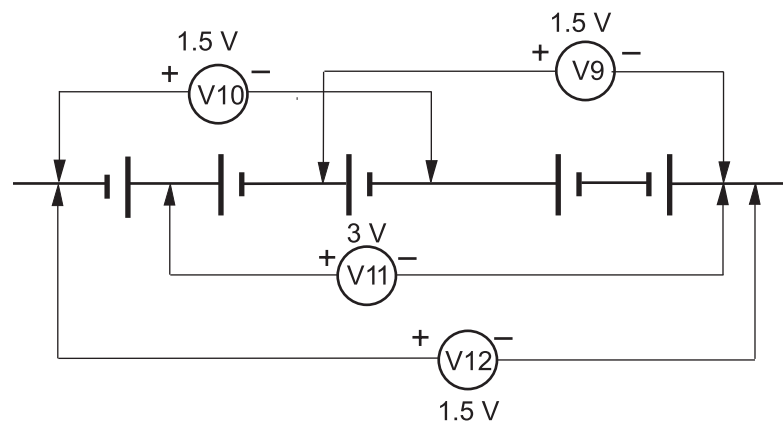


Figure 3

Self-Test 4

1. c. 2
2. circuit d
3. yes
4. 6 A
5. d. current increases
6. a. $E_T = E_1 = E_2 = E_3$
7. a. 2 A
b. 3 A
c. 6 A
8. b. decrease and the total circuit resistance will increase

Self-Test 5

1. $10\ \Omega$
2. a. $E_1 = 200\text{ V}$
b. $E_2 = 200\text{ V}$
c. $E_T = 200\text{ V}$
d. $I_2 = 5\text{ A}$
e. $R_T = 25\ \Omega$
f. $P_T = 1600\text{ W}$

3. $0.0025\ \Omega$

Solution:

$$I_T = 20\text{ A}$$

$$I_m = 1\text{ mA} = 0.001\text{ A}$$

$$\begin{aligned} I_s &= 20\text{ A} - 0.001\text{ A} \\ &= 19.999\text{ A} \end{aligned}$$

$$\frac{R_m}{R_s} = \frac{I_s}{I_m}$$

$$\frac{50 \Omega}{R_s} = \frac{19.999 \text{ A}}{0.001 \text{ A}}$$

$$R_s = 0.0025 \Omega$$

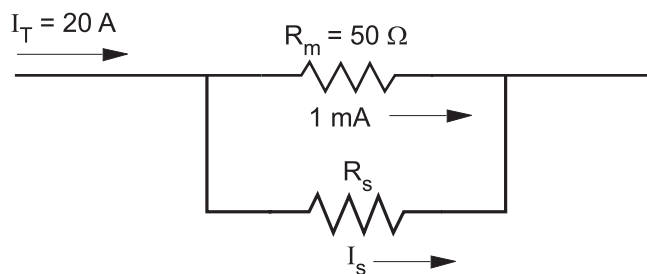


Figure 1—Circuit for Question 3

4. Because of the number of values you have to find, create a table: Write in the known values, then calculate and write in the required values, which are shown below in brackets.

Table 1 Circuit value chart for Question 4

	E	I	R	P
Branch 1	(250 V)	50 A	(5 Ω)	
Branch 2	250 V	(25 A)	10 Ω	
Branch 3	250 V	12.5 A	(20 Ω)	
Branch 4	250 V	(4.167 A)	60 Ω	
Branch 5	250 V	8.333 A	(30 Ω)	(2083 W)
Total	(250 V)	100 A	(2.5 Ω)	(25 kW)

5. c. 20 Ω
- 6.

Table 1 Circuit value chart for Question 6

	E	I	R	P
Branch 1	120 V	0.2 A	600 Ω	24 W
Branch 2	120 V	0.4 A	300 Ω	48 W
Branch 3	120 V	0.6 A	200 Ω	72 W
Total	120 V	1.2 A	100 Ω	144 W

7. a. $E_2 = 16\text{ V}$
 b. $E_T = 16\text{ V}$
 c. $R_1 = 8\ \Omega$

8. $12\ \Omega$

9. $P_1 = 720\text{ W}$
 $P_2 = 240\text{ W}$
 $P_3 = 720\text{ W}$
 $P_T = 1680\text{ W}$

10. 2.5 A

Self-Test 6

1. a. 24 V
 b. 0 V
2. Power can still be delivered to the load at rated voltage with any one of the power sources removed.
- 3.

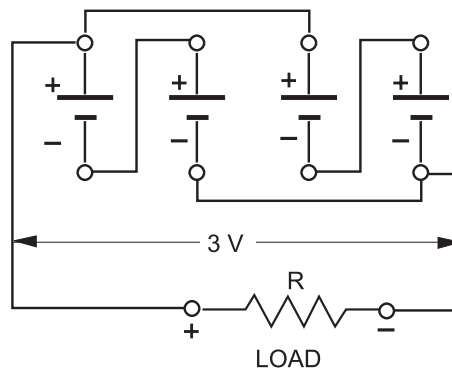


Figure 1—Circuit for Question 3

4. 4 A

Self-Test 7

1. b. parallel circuit
2. a. $R_5 = 8 \text{ A}$
b. $R_2 = 2 \text{ A}$
c. $R_3 = 4 \text{ A}$
- 3.

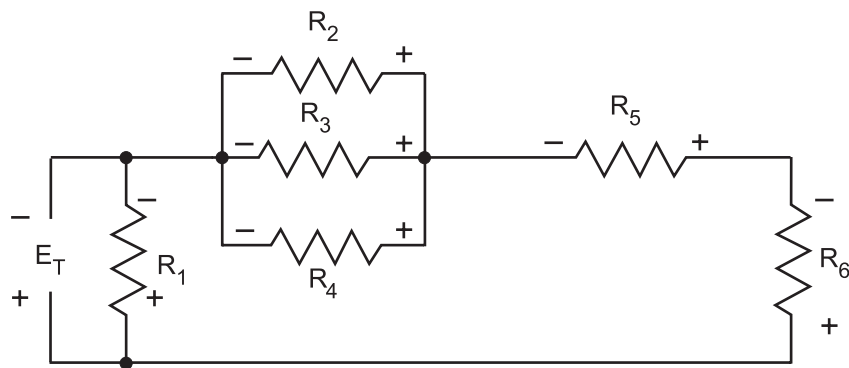


Figure 1—Circuit for Question 3 with correct polarities marked

Self-Test 8

1. a. $R_T = 16 \Omega$
b. $I_T = 7.5 \text{ A}$
c. $E_1 = 45 \text{ V}$
 $E_2 = E_3 = 15 \text{ V}$
 $E_4 = E_5 = 30 \text{ V}$
 $E_6 = 30 \text{ V}$
d. 900 W

2.

Table 1 Circuit value chart for Question 2

	E	I	R
Resistor 1	(30 V)	(3 A)	10 Ω
Resistor 2	(20 V)	(1 A)	20 Ω
Resistor 3	(30 V)	(1 A)	30 Ω
Resistor 4	(30 V)	(2 A)	15 Ω
Resistor 5	(20 V)	(2 A)	10 Ω
Total	80 V	(3 A)	(26.7 Ω)

3. 2 Ω

In series, each resistor (R_{each}) is $R_{\text{each}} = 72 \Omega \div 6 = 12 \Omega$.

Therefore, in parallel, $R_T = 12 \Omega \div 6 = 2 \Omega$.

4. a. $E_T = 120 \text{ V}$ b. $I_3 = 8 \text{ A}$ c. $R_1 = 30 \Omega$ d. $R_2 = 10 \Omega$ e. $R_3 = 15 \Omega$

First, solve for I_T using Kirchhoff's current law at node A in Figure 2:

$$\begin{aligned}
 I_T &= I_1 + I_A \\
 &= 4 \text{ A} + 20 \text{ A} = 24 \text{ A}
 \end{aligned}$$

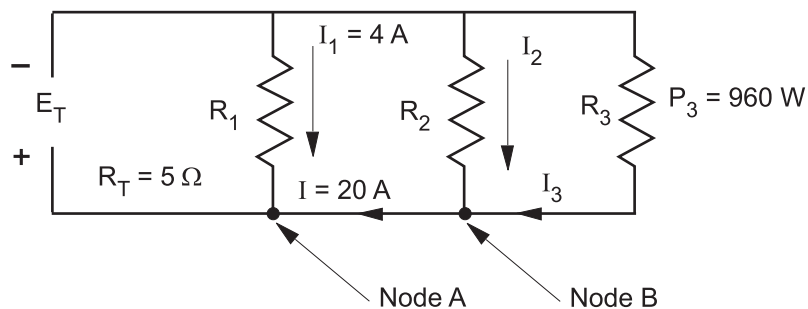


Figure 2—Circuit showing nodes A and B

Now apply Ohm's law to solve for E_T .

$$\begin{aligned}E_T &= I_T \times R_T \\&= 24 \text{ A} \times 5 \Omega \\&= 120 \text{ V}\end{aligned}$$

Notice also that $E_T = E_1 = E_2 = E_3$.

Now you can solve for I_3 using the power equation $P_3 = E_3 \times I_3$:

$$\begin{aligned}I_3 &= P_3 \div E_3 \\&= 960 \text{ W} \div 120 \text{ V} \\&= 8 \text{ A}\end{aligned}$$

Solve for R_1 using Ohm's law:

$$\begin{aligned}R_1 &= E_1 \div I_1 \\&= 120 \text{ V} \div 4 \text{ A} = 30 \Omega\end{aligned}$$

Solve for I_2 using Kirchhoff's current law at node B in Figure 2:

$$\begin{aligned}I_B &= I_2 + I_3 \\I_2 &= 20 \text{ A} - 8 \text{ A} = 12 \text{ A}\end{aligned}$$

Now use Ohm's law to find R_2 :

$$\begin{aligned}R_2 &= E_2 \div I_2 \\&= 120 \text{ V} \div 12 \text{ A} = 10 \Omega\end{aligned}$$

Apply Ohm's law with the previously solved values to find R_3 :

$$\begin{aligned}R_3 &= E_3 \div I_3 \\&= 120 \text{ V} \div 8 \text{ A} = 15 \Omega\end{aligned}$$

5. $E_1 = 240 \text{ V}$
 $E_2 = 210 \text{ V}$
 $E_3 = 50 \text{ V}$
 $E_4 = 35 \text{ V}$
 $E_5 = 70 \text{ V}$
 $E_6 = 55 \text{ V}$

First, find all of the individual current values, from which you can determine the IR drops across the individual resistors.

Mark the polarity for each resistor, as shown in Figures 4a and 4b below. This shows the direction of the currents (from negative to positive).

In Figure 3a, the currents are determined as follows:

$$I_T = 10\text{ A} + 5\text{ A} = 15\text{ A}$$

From Kirchhoff's current law,

$$I_a + I_b = 5\text{ A}$$

$I_a = I_b$ since each of the parallel resistors are equal.

So, $I_a = 2.5\text{ A}$ and $I_b = 2.5\text{ A}$.

Using Kirchhoff's current law again,

$$5\text{ A} = 4\text{ A} + I_g, \text{ so therefore } I_g = 1\text{ A}.$$

You now have the current value for each resistor in the circuit; so using Ohm's law, you can calculate each voltage drop as marked in Figure 3b.

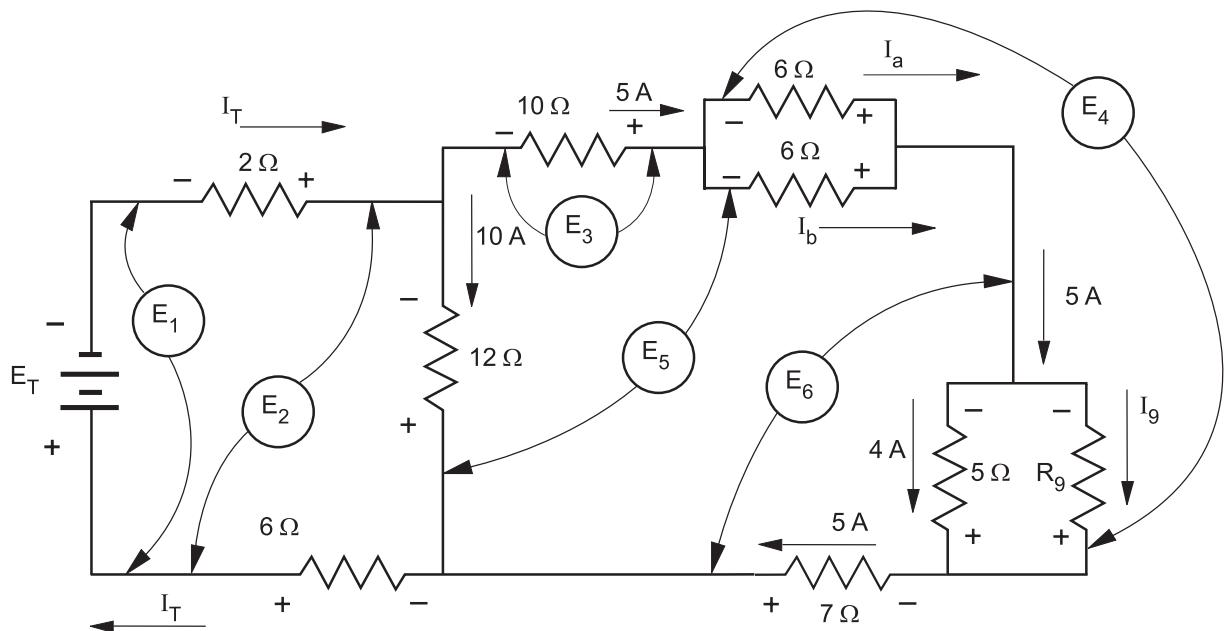


Figure 3a—Currents in the circuit for Question 5

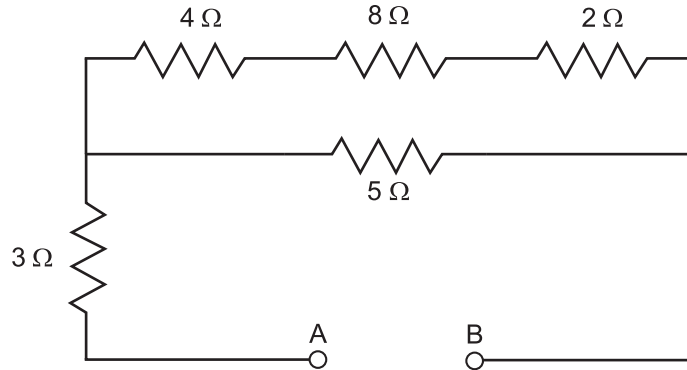


Figure 4a—Circuit between points A and B for Question 6a

For the parallel portion of Figure 4a, you should see three series resistors ($4\ \Omega + 8\ \Omega + 2\ \Omega = 14\ \Omega$) in parallel with $5\ \Omega$.

Using the product and sum method, this gives an equivalent resistance of:

$$R_p = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{14 \times 5}{14 + 5} = \frac{70}{19} = 3.68\ \Omega$$

This equivalent parallel resistance is in series with $3\ \Omega$ to give a total circuit resistance of:

$$R_T = 3.68 + 3 = 6.68\ \Omega$$

- b. Between points B and C, the circuit appears as shown in Figure 4b.

The calculation method is similar to that used in Question 6a.

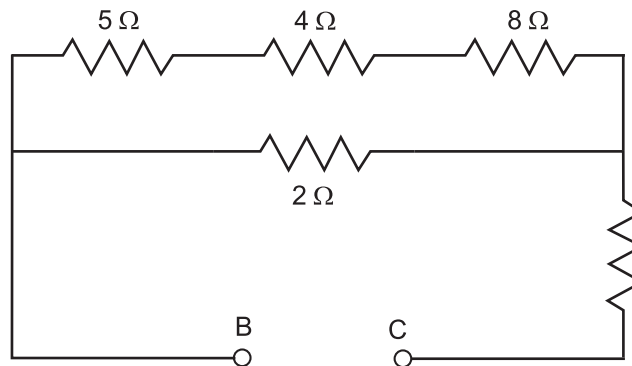


Figure 4b—Circuit between points B and C for Question 5b

- c. Between points A and C, the circuit appears as shown in Figure 4c. The calculation method is similar to that used in Questions 6a and 6b.

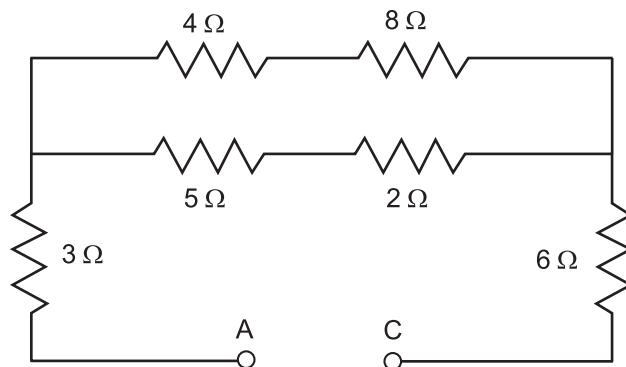


Figure 4c—Circuit between points A and C for Question 6c

7. a. $E_T = 120 \text{ V}$
 b. $I_T = 10 \text{ A}$
 c. $R_T = 12 \Omega$
 d. $R_3 = 8.57 \Omega$

Given various values of current, the first step may be to try to determine the currents through each resistor by applying Kirchhoff's current law.

Referring to Figure 5a on the next page:

Given that the current through the "top branch" of Figure 5a is 4 A, find the current through R_3 :

$$I_3 = 4 - 0.5 = 3.5 \text{ A}$$

In the "lower branch" of Figure 5a, there is a current of 2 A through the 20Ω resistor. Applying Ohm's law, the voltage drop across this resistor is 40 V.

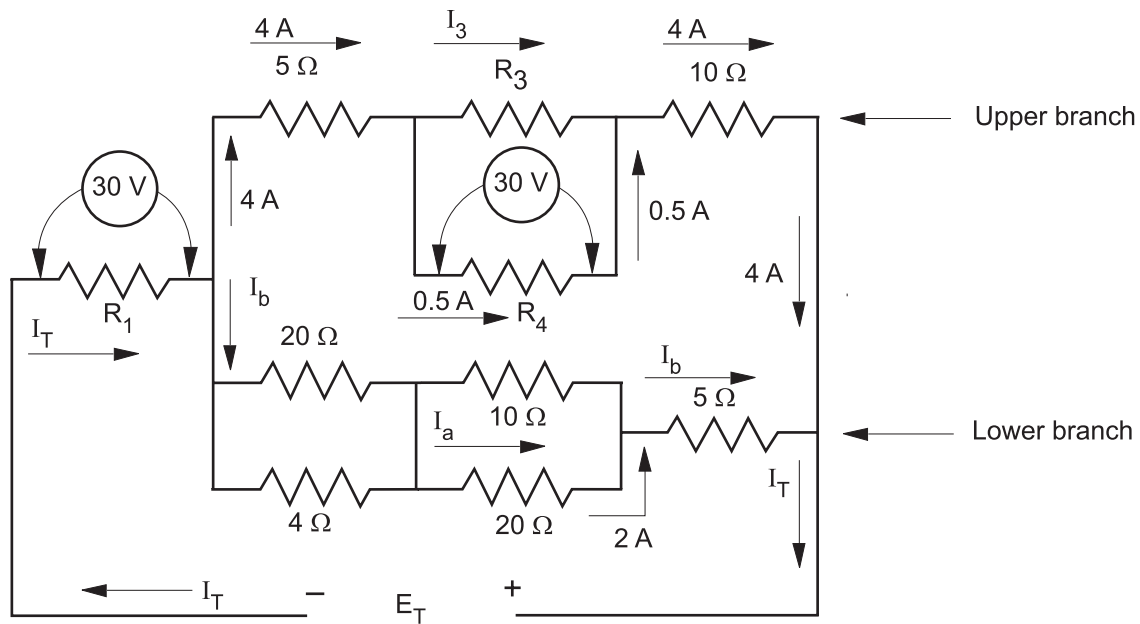


Figure 5a—Currents in the circuit for Question 7

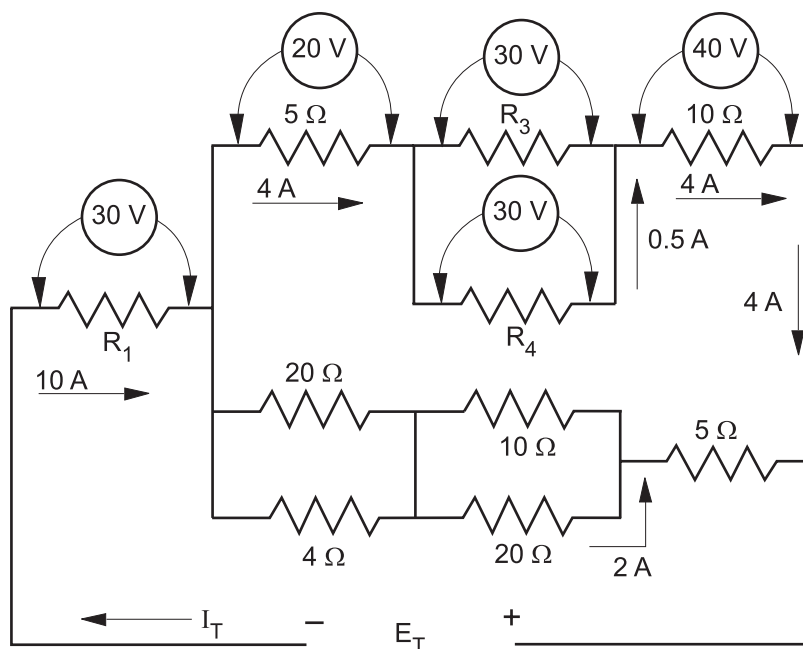


Figure 5b—Voltages in the circuit for Question 7

Since this $20\ \Omega$ resistor is in parallel with a $10\ \Omega$ resistor, use the $40\ \text{V}$ drop across them to solve for the current I_a through the $10\ \Omega$ resistor:

Applying Ohm's law, $I_a = 4\ \text{A}$.

Now, solve for the current I_b :

$$I_b = I_a + 2 \text{ A} = 4 \text{ A} + 2 \text{ A} = 6 \text{ A}$$

The total circuit current is the sum of the “upper” and “lower” branch currents.

$$I_T = 4 \text{ A} + I_b = 4 \text{ A} + 6 \text{ A} = 10 \text{ A}, \text{ which is the answer to part “b” of this problem.}$$

Now, determine the total circuit voltage by summing all of the IR drops in a current loop.

Start with the voltage drop across R_1 and move to the “upper” branch and add the IR drop of the 5Ω resistor plus the 30 V drop across R_3 plus the IR drop across the 10Ω resistor.

$$E_T = 30 \text{ V} + 20 \text{ V} + 30 \text{ V} + 40 \text{ V}$$

$$= 120 \text{ V}, \text{ which is the answer to part “a” of this problem.}$$

With the total voltage and total current solved, now use Ohm’s law to find the total resistance:

$$R_T = E_T \div I_T = 120 \text{ V} \div 10 \text{ A} = 12 \Omega, \text{ which is the answer to part “c”}.$$

Finally, calculate the value of resistor R_3 using Ohm’s law:

$$R_3 = 30 \text{ V} \div 3.5 \text{ A} = 8.57 \Omega$$

Self-Test 9

1. a. +120 V
b. +100 V
c. +60 V
2. a. +60 V
b. +40 V
c. -60 V
3. a. 6 V
b. 0 V
c. 12 V
4. 4 V

Self-Test 10

1. $B_1 = \text{positive}$
 $B_2 = \text{positive}$
 $B_3 = \text{negative}$
 $B_4 = \text{negative}$
2. $B_1 = +280 \text{ V}$
 $B_2 = +120 \text{ V}$
 $B_3 = -60 \text{ V}$
 $B_4 = -100 \text{ V}$
3. $B_1 = +268 \text{ V}$
 $B_2 = +89 \text{ V}$
 $B_3 = -67 \text{ V}$
 $B_4 = -112 \text{ V}$

Self-Test 11

1. the same
2. lower to higher
3. difference
4. voltage
5. 24 V
6. right to left
7. $R_x = \frac{R_2 \times R_3}{R_1}$

Self-Test 12

1. d. precise resistance
2. zero
3. to prevent battery drain when the instrument is not in use

4. $R_x = \frac{R_1}{R_2} \times R_3$

5. 1.17Ω

6. 2560Ω

Self-Test 13

1. a. The two outer conductors are called line (or hot) conductors; the common conductor is called the neutral.
b. The neutral is white or natural grey; the line conductors are black or sometimes one black and one red.
2. so there will be close to zero current in the neutral
3. If the loads are unbalanced, there could be an increase in voltage across some of the higher resistance loads because they are now connected in series across 240 V. This could cause damage due to overvoltage.
4. 3 A
5. If the fuse blows, the neutral is opened. If the circuit is unbalanced, an overvoltage can result.
6. a. 25 A
b. 20 A
c. 5 A

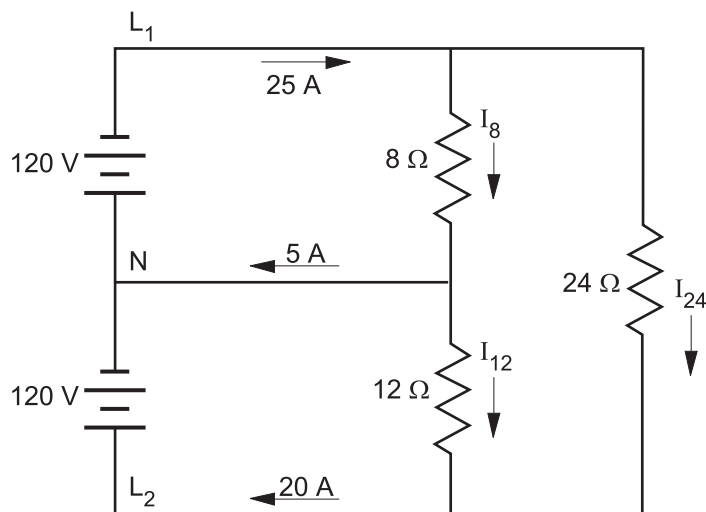


Figure 1—Circuit for calculating Question 6

$$I_8 = 120 \text{ V} \div 8 \Omega = 15 \text{ A}$$

$$I_{12} = 120 \text{ V} \div 12 \Omega = 10 \text{ A}$$

$$I_{24} = 240 \text{ V} \div 24 \Omega = 10 \text{ A}$$

Therefore,

$$I_{L1} = 15 \text{ A} + 10 \text{ A} = 25 \text{ A}$$

$$I_{L2} = 10 \text{ A} + 10 \text{ A} = 20 \text{ A}$$

$$I_N = 25 \text{ A} - 20 \text{ A} = 5 \text{ A}$$

Self-Test 14

1. $V_{L1} = 105 \text{ V}$

$$V_{L2} = 96 \text{ V}$$

$$V_{L3} = 117 \text{ V}$$

$$V_{L4} = 117 \text{ V}$$

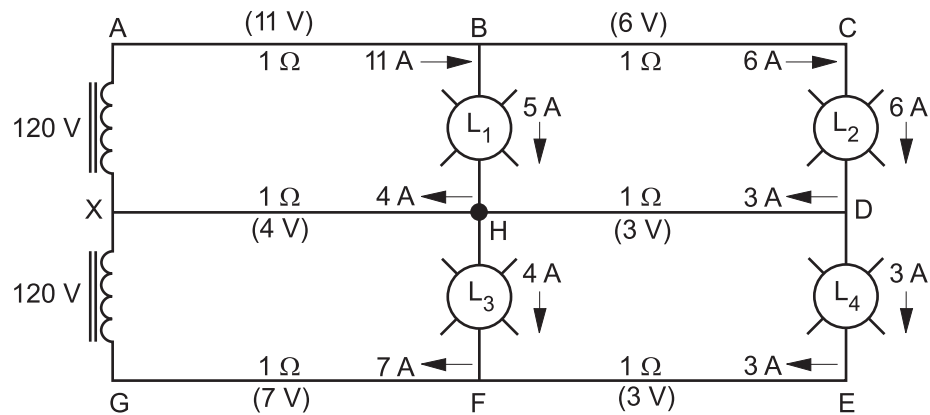


Figure 1—Circuit for calculating Question 1

Applying Kirchhoff's current law at point A,

$$I_{AB} = 6 \text{ A} + 5 \text{ A} = 11 \text{ A}$$

Applying Ohm's law to line AB,

$$V_{AB} = 11 \text{ A} \times 1 \Omega = 11 \text{ V}$$

Similarly,

$$I_{BC} = 6 \text{ A} \text{ and } V_{BC} = 6 \text{ V}$$

$$I_{DH} = 3 \text{ A} \text{ and } V_{DH} = 3 \text{ V}$$

$$I_{HX} = 4 \text{ A} \text{ and } V_{HX} = 4 \text{ V}$$

$$I_{EF} = 3 \text{ A} \text{ and } V_{EF} = 3 \text{ V}$$

$$I_{FG} = 7 \text{ A} \text{ and } V_{FG} = 7 \text{ V}$$

$$V_{L1} = 120 - 11 - 4 \text{ V} = 105 \text{ V}$$

$$V_{L2} = 105 - 6 - 3 \text{ V} = 96 \text{ V}$$

$$V_{L3} = 120 + 4 - 7 \text{ V} = 117 \text{ V}$$

$$V_{L4} = 117 + 3 - 3 \text{ V} = 117 \text{ V}$$

$$2. \quad V_{L1} = 120 - 9 + 2 \text{ V} = 113 \text{ V}$$

$$V_{L2} = 113 - 7 - 2 \text{ V} = 104 \text{ V}$$

$$V_{L3} = 120 - 2 - 11 \text{ V} = 107 \text{ V}$$

$$V_{L4} = 107 + 2 - 5 \text{ V} = 104 \text{ V}$$

These calculations are done exactly as in Question 1. Intermediate values are shown in Figure 2.

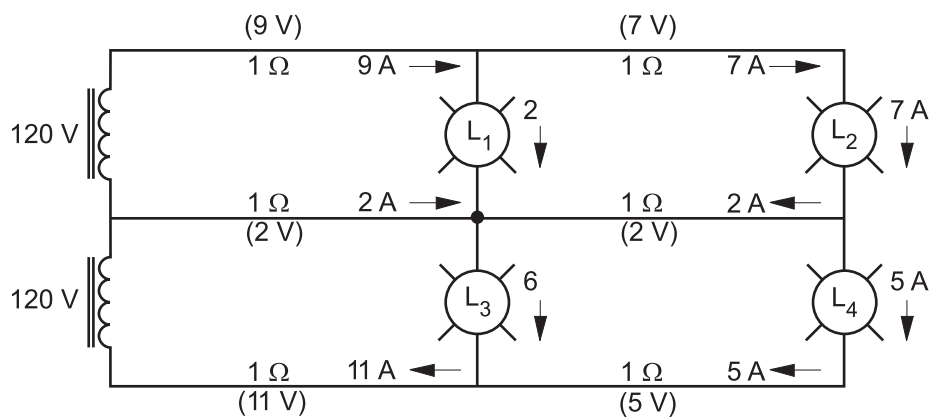


Figure 2—Circuit for calculating Question 2

$$3. \quad V_{L1} = 120 - 10 \text{ V} = 110 \text{ V}$$

$$V_{L2} = 110 - 6 \text{ V} = 104 \text{ V}$$

$$V_{L3} = 120 - 10 \text{ V} = 110 \text{ V}$$

$$V_{L4} = 110 - 6 \text{ V} = 104 \text{ V}$$

These calculations are done exactly as in Questions 1 and 2. Intermediate values are shown in Figure 3.

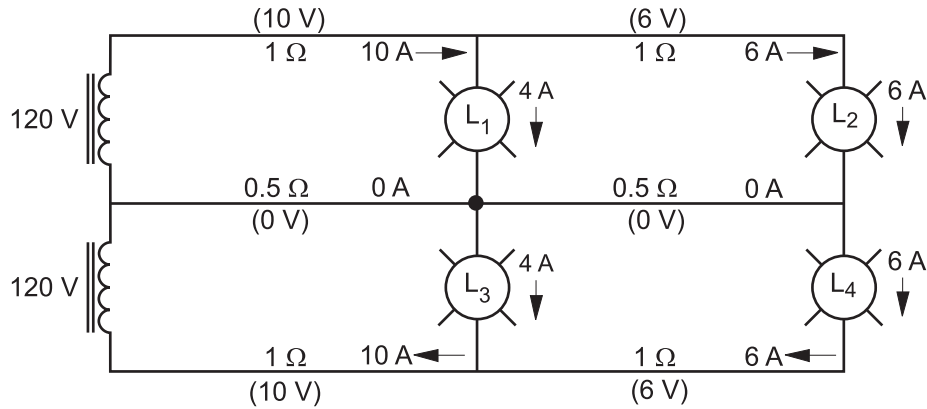


Figure 3—Circuit for calculating Question 3

4. a. 57.38 V
 b. 155.95 V
 c. 4.14 A
 d. 16.56 V
 e. 49.3 V

First find the resistance values
 of loads 1 and 2.

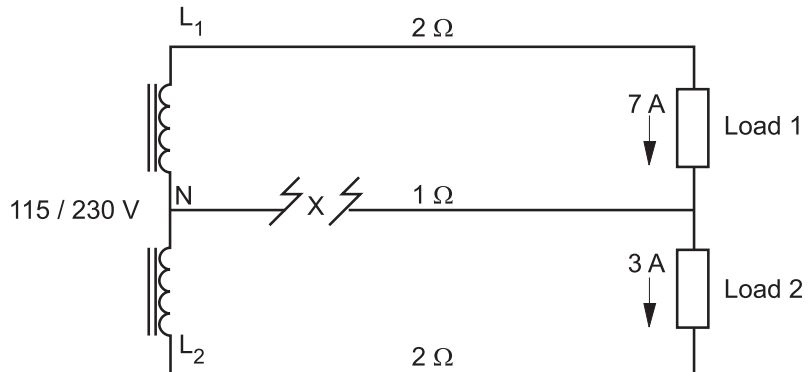
With neutral closed:

$$\begin{aligned} V_{L1} &= 115 - 14 - 4 \text{ V} = 97 \text{ V} \\ R_1 &= 97 \text{ V} \div 7 \text{ A} = 13.86 \Omega \\ V_{L2} &= 115 + 4 - 6 \text{ V} = 113 \text{ V} \\ R_2 &= 113 \text{ V} \div 3 \text{ A} = 37.67 \Omega \end{aligned}$$

Then calculate the required values.

With neutral open:

$$\begin{aligned} \text{Total circuit voltage} &= 230 \text{ V} \\ \text{Total circuit resistance} &= 2 + 13.86 + 37.76 + 2 \Omega = 55.53 \Omega \\ \text{Total line current} &= 230 \text{ V} \div 55.53 \Omega = 4.14 \text{ A} \\ \text{Voltage across load 1} &= 4.14 \text{ A} \times 13.86 \Omega = 57.38 \text{ V} \\ \text{Voltage across load 2} &= 4.14 \text{ A} \times 37.67 \Omega = 155.95 \text{ V} \\ \text{Total line drop} &= 4.14 \text{ A} \times 2 = 16.56 \text{ V} \\ \text{Voltage across break} &= 115 - 8.28 - 57.38 \text{ V} = 49.3 \text{ V using load 1} \\ \text{or} &= 8.28 + 155.95 - 115 \text{ V} = 49.3 \text{ V using load 2} \end{aligned}$$

Figure 4—Redrawn circuit for upper loop,
load 1, to calculate Question 4e

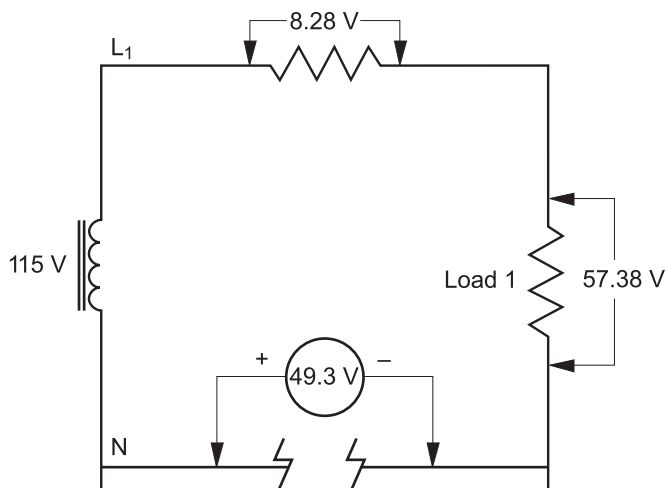


Figure 5—Circuit with neutral closed for Question 5

5. a. 109.1 V
 b. 109.1 V
 c. 130.9 V
 d. 130.9 V
 e. 10.91 A
 f. 10.91 V

First find the equivalent resistance values of loads 1//2 and of loads 3//4.

With neutral closed:

$$V_{L1} = V_{L2} = V_{L3} = V_{L4} = 120 \text{ V}$$

$$\text{Resistance of } L_1 \text{ and } L_2 \text{ in parallel} = 120 \text{ V} \div 12 \text{ A} = 10 \Omega$$

$$\text{Resistance of } L_3 \text{ and } L_4 \text{ in parallel} = 120 \text{ V} \div 10 \text{ A} = 12 \Omega$$

Then calculate the required values.

With neutral open:

$$\text{Total circuit voltage} = 240 \text{ V}$$

$$\text{Total line current} = 240 \text{ V} \div 22 \Omega = 10.91 \text{ A}$$

$$\begin{aligned} \text{Voltage across load 1} &= \text{Voltage across load 2} \\ &= 10.91 \text{ A} \times 10 \Omega = 109.1 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Voltage across load 3} &= \text{Voltage across load 4} \\ &= 10.91 \text{ A} \times 12 \Omega = 130.9 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Voltage across break} &= 120 - 109.1 \text{ V} = 10.9 \text{ V using load 1} \\ \text{or} &= 130.9 - 120 \text{ V} = 10.9 \text{ V using load 2} \end{aligned}$$

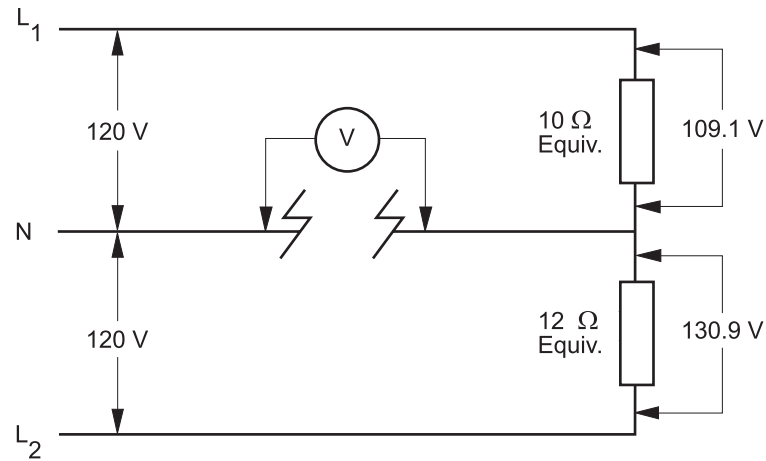


Figure 6—Equivalent circuit for Question 5

6. a. 120 V
 b. 144 V
 c. 120 V
 d. 96 V
 e. circuit is unbalanced:
 10.8 A in line 1
 8.8 A in line 2
 2 A in neutral to source
 f. 24 V

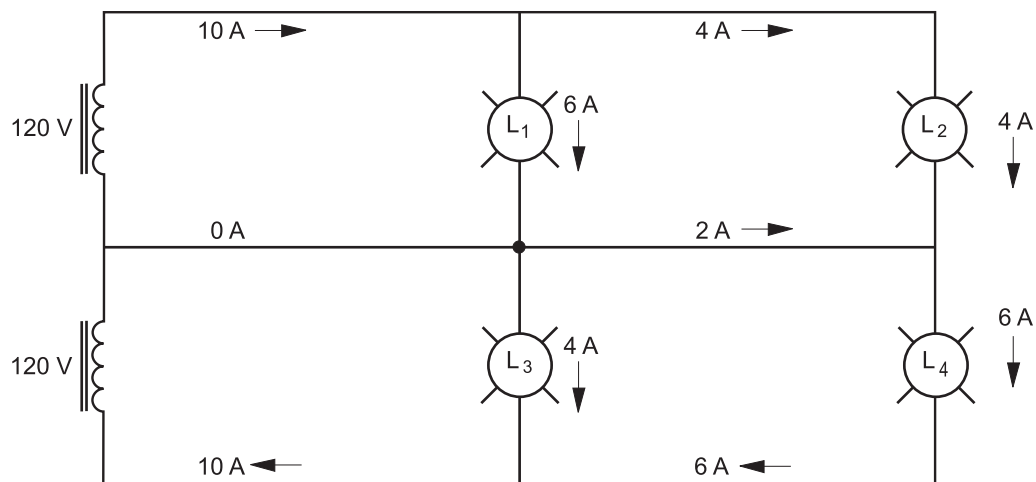


Figure 7—Circuit with neutral closed for Question 6

First find the resistance values

With neutral closed:

$$R_1 = 120 \text{ V} \div 6 \text{ A} = 20 \Omega$$

$$R_2 = 120 \text{ V} \div 4 \text{ A} = 30 \Omega$$

$$R_3 = 120 \text{ V} \div 4 \text{ A} = 30 \Omega$$

$$R_4 = 120 \text{ V} \div 6 \text{ A} = 20 \Omega$$

Then calculate the required values

With neutral open:

Current in load 1 = as before

$$= 120 \text{ V} \div 20 \Omega = 6 \text{ A}$$

Current in load 3 = as before

$$= 120 \text{ V} \div 30 \Omega = 4 \text{ A}$$

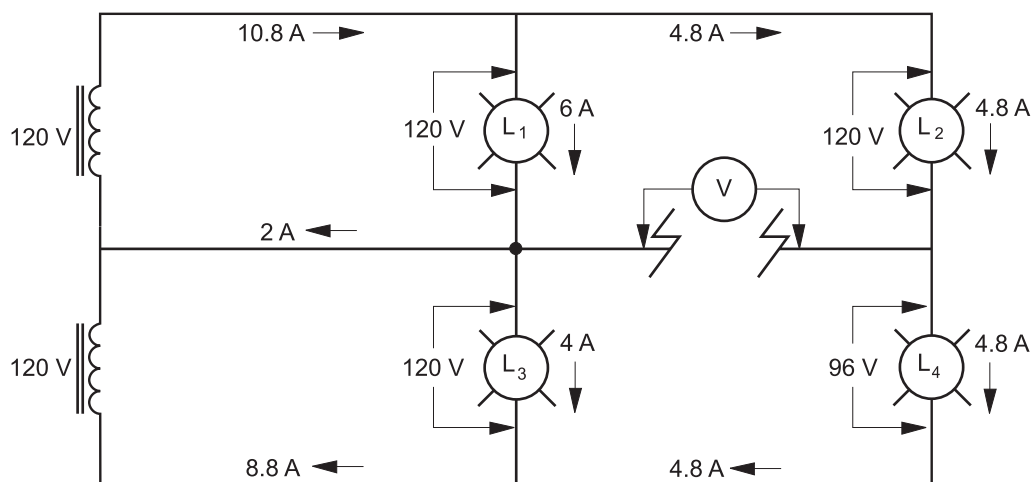


Figure 8—Circuit with neutral open for Question 6

Loads 2 and 4 are in series across 240 V

$$\text{Current in loads 2 and 4} = 240 \text{ V} \div 50 \Omega = 4.8 \text{ A}$$

$$\text{Total current in line 1} = 6 \text{ A} + 4.8 \text{ A} = 10.8 \text{ A}$$

$$\text{Total current in line 2} = 4 \text{ A} + 4.8 \text{ A} = 8.8 \text{ A}$$

$$\begin{aligned} \text{Current in neutral to} \\ \text{source line} &= 10.8 \text{ A} - 8.8 \text{ A} = 2 \text{ A} \end{aligned}$$

$$\text{Voltage across load 1} = 120 \text{ V, as before}$$

$$\text{Voltage across load 3} = 120 \text{ V, as before}$$

$$\begin{array}{llll} \text{Voltage across load 2} & = & 4.8 \text{ A} \times 30 \, \Omega & = 144 \text{ V} \\ \text{Voltage across load 4} & = & 4.8 \text{ A} \times 20 \, \Omega & = 96 \text{ V} \\ \text{Voltage across break} & = & 120 \text{ V} - 96 \text{ V} & = 24 \text{ V} \end{array}$$

