CONSTRUCTION ELECTRICIAN APPRENTICESHIP PROGRAM

Level 3

Line D: Apply Circuit Concepts



LEARNING GUIDE D-5

ANALYZE THREE-PHASE CIRCUITS

ita
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THE RIGHT SKILLS ▶ A PROVEN ADVANTAGE

BRITISH COLUMBIA

Foreword

The Industry Training Authority (ITA) is pleased to release this major update of learning resources to support the delivery of the BC Electrician Apprenticeship Program. It was made possible by the dedicated efforts of the Electrical Articulation Committee of BC (EAC).

The EAC is a working group of electrical instructors from institutions across the province and is one of the key stakeholder groups that supports and strengthens industry training in BC. It was the driving force behind the update of the Electrician Apprenticeship Program Learning Guides, supplying the specialized expertise required to incorporate technological, procedural and industry-driven changes. The EAC plays an important role in the province's post-secondary public institutions. As discipline specialists the committee's members share information and engage in discussions of curriculum matters, particularly those affecting student mobility.

ITA would also like to acknowledge the Construction Industry Training Organization (CITO) which provides direction for improving industry training in the construction sector. CITO is responsible for organizing industry and instructor representatives within BC to consult and provide changes related to the BC Construction Electrician Training Program.

We are grateful to EAC for their contributions to the ongoing development of BC Construction Electrician Training Program Learning Guides (materials whose ownership and copyright are maintained by the Province of British Columbia through ITA).

Industry Training Authority
January 2011

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ANALYZE THREE-PHASE CIRCUITS

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Learning Objectives

- The learner will be able to describe the characteristics of three-phase AC circuits.
- The learner will be able to calculate voltage, current, impedance, power and power factor in three-phase AC circuits.
- The learner will be able to apply power factor correction to three-phase AC circuits.

Activities

- Read and study the topics of Learning Guide D-5: Analyze Three-Phase Circuits
- Complete Self-Tests 1 through 9. Check your answers with the Answer Key provided at the end of this Learning Guide.

Resources

You are encouraged to obtain the following text to provide supplemental learning information:

Alternating Current Fundamentals, 8th edition by Stephen L. Herman, Delmar Cengage Learning

BC Trades Modules
www.bctradesmodules.ca

We want your feedback! Please go the BC Trades Modules website to enter comments about specific section(s) that require correction or modification. All submissions will be reviewed and considered for inclusion in the next revision.

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Industry Training Authority
January 2011

Learning Task 1:

Describe the characteristics of three-phase AC

Advantages of three-phase systems

So far, you have learned about single-phase systems. This is the easiest way to start to learn about circuit concepts and analysis. However, three-phase (also written as 3ϕ or 3-phase) systems are widely used for the generation, transmission and distribution of electric power because they are superior to single-phase systems in many ways.

Some of these advantages are:

- It is possible to produce smaller machines with the same kVA capacity. A three-phase generator or motor built in the same frame size as a single-phase motor can have a kVA rating of about 1.5 times that of the single-phase machine.
- A three-phase distribution system requires about ¾ as much copper to distribute the same energy with the same efficiency at the same voltage as is required by a single-phase system.
- A three-phase system produces three sine waves spaced 120 electrical degrees apart. Because of this, the power delivered to a balanced three-phase load is constant at all times, while the power delivered to a single-phase load is pulsating. This means that the operating characteristics of three-phase machines are far superior to similar single-phase machines.
- The design of three-phase motors is much simpler than single-phase motors. Three-phase motors are self-starting and do not require a starting winding.

Generation of three-phase voltages

You will recall that a single-phase alternator generates a sine-wave voltage when the armature conductors are cut by a rotating magnetic field. If a second set of windings is placed on the armature of an alternator, displaced from the first set by some number of electrical degrees, a second alternating voltage will be produced by the alternator. This second sine-wave voltage will reach its peaks at different times than the voltage produced by the first winding set.

Each of the two sine wave voltages is referred to as a *phase*. Polyphase systems are systems that have two or more phases. The most common polyphase system is the three-phase system.

Three-phase alternators are constructed with three separate sets of windings placed on the armature 120 electrical degrees apart. For simplicity, each winding set in Figure 1 contains only one loop. As the field rotates it produces three sine waves of equal frequency and magnitude, all out of phase with each other by 120 degrees.

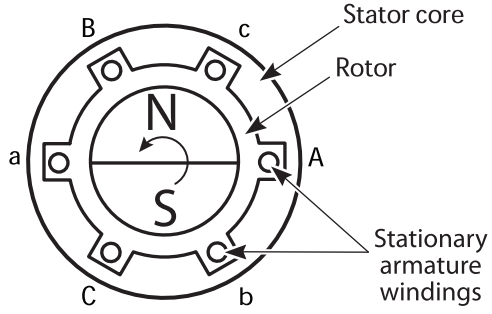


Figure 1—Three-phase alternator

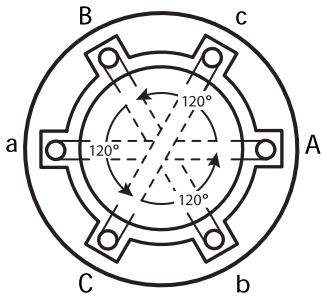


Figure 2—Stationary armature windings

Notice that in this two-pole alternator, the two sides of each phase winding are 180° apart. For example, the winding labelled A to a enters the diagram at the point marked A, crosses behind the diagram and comes out of the diagram at the point marked a. (Figure 2). This means that the voltage measured between the two ends of this winding will equal the sum of the voltage induced in the A side plus the voltage induced in the a side.

The voltage induced in each winding depends on the rate of change of flux surrounding the armature conductor. This voltage peaks as the centre of the rotor pole is adjacent to the armature winding.

Figure 3 shows the position of the rotor when it produces the positive peaks in voltage of the three different phases. Follow the vertical dotted line down the graph to see where in their cycle the other two phases are when the one phase is at its positive peak. A vertical line drawn anywhere on the graph can be used to compare the voltages of all three phases relative to each other at any instant.

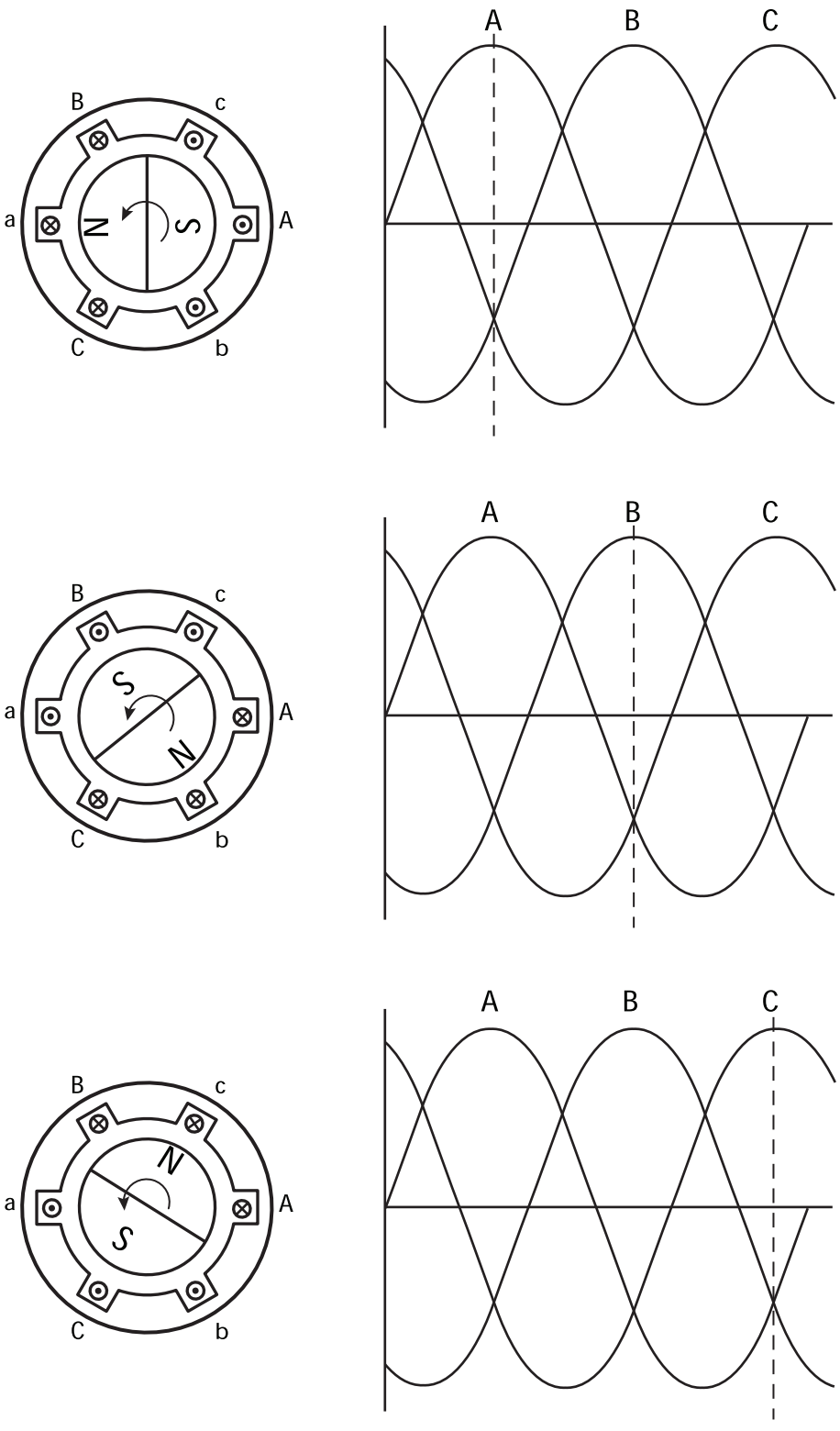


Figure 3—Rotor position for the positive peaks of induced voltage

Wye (Y) and delta (Δ) connections

The three single-turn armature windings in the alternator of Figure 1 are oriented at 120° to each other. If we neglect the other parts of the alternator and show the physical orientation of three multi-turn windings in a similar three-phase alternator, it would look something like Figure 4.

Bringing six leads out from the alternator to feed three separate single-phase circuits would defeat the advantages of the three-phase system.

The windings of a three-phase alternator are generally brought out in one of two configurations: the Y (or star) configuration or the Δ configuration. "Y" is sometimes written as wye. "Δ" is sometimes written as delta.

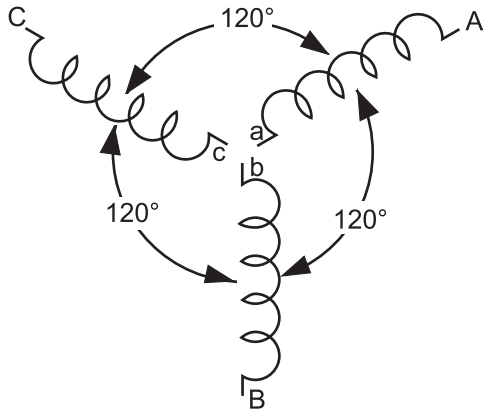


Figure 4—Three-phase winding orientation

Y-connected windings

If the three ends of the windings labelled with lower-case letters are tied together at a common point, called the star point, the alternator is said to be Y-connected. Figure 5 shows this configuration with the star point labelled N. An alternator connected in this configuration has four armature leads. The lead from the star point is the neutral and is labelled N. The other three leads are the line leads and are labelled A, B and C.

Δ-connected windings

The other common way of internally connecting the leads of the alternator is the Δ configuration. Figure 6 shows a schematic representation of this configuration.

The lower-case end of the B winding is connected to the upper-case end of the A winding. Next, the lower-case end of the C winding is connected to the upper-case end of the B winding. Finally, the lower-case end of the A winding is connected to the upper-case end of the C winding. This forms a complete armature loop. One lead is attached to each junction and brought out of the alternator. These three line leads are labelled A, B and C.

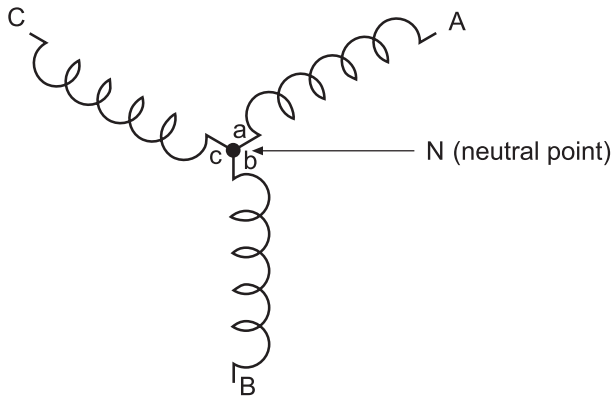


Figure 5—Y configuration

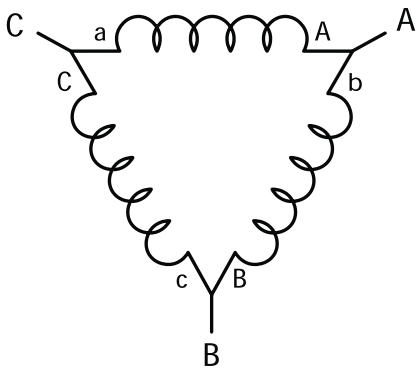


Figure 6—Δ configuration

Phase sequence

The order in which the three phases reach their positive peaks is referred to as the *phase sequence*. The two phase sequences are either A-B-C or C-B-A. The phase sequence can be reversed by one of two methods:

- The direction of the prime mover can be reversed so that the rotor turns in the other direction.
- Any two of the line leads may be interchanged. For example, the lead labelled A could be interchanged with the lead labelled B so that B becomes A and A becomes B. This could also be done by interchanging A and C or B and C.

The Canadian Electrical Code requires that the three phases be identified by colour coding. The colours are as follows:

- Phase A = Red
- Phase B = Black
- Phase C = Blue
- Neutral = White or natural grey

Phasor representation of three-phase voltages and currents

Earlier you learned about phasors. Phasors are rotating vectors used to represent quantities that vary with time. Sinusoidal voltages and currents are quantities whose magnitudes vary with time, so they may be represented by phasors.

Remember there are certain conventions (see Figure 7) that must be observed when working with phasors. These are:

- The length of the phasor is most often used to represent the effective value of the sine wave, but at times it is drawn to represent the peak value of the sine wave.
- Phasor algebra is done the same way as vector algebra.
- The positive direction of phasor rotation is counter-clockwise from the zero degree reference at 3 o'clock.
- When two phasors are shown on the same phasor diagram, the phasor that is *furthest counter-clockwise* is said to *lead* the second phasor.
- The waveforms of phasors in the same phasor diagram must be sinusoidal and have the same frequency. This is because when two sine waves of the same frequency are added, the result is always a third sine wave of the same frequency.
- To draw phasors, they must be frozen in time. Normally, one of the phasors will be picked as the reference phasor and drawn on the reference axis at 0° or 3 o'clock. This makes working with the phasors less complicated.
- In series circuits, the current is the same everywhere in the circuit, so the current phasor is normally drawn on the reference axis.
- In parallel circuits, it is the voltage that is the same everywhere in the circuit, so the voltage vector is normally drawn on the reference axis.
- In three-phase circuits, phase A is generally used as the reference phasor.
- Similar sinusoidal waveforms on the same phasor diagram must be drawn to the same scale. (For example, two voltage waveforms, two current waveforms or two power waveforms on the same diagram.) However, if two dissimilar sinusoidal waveforms are drawn on the same diagram, their scales may differ. (For example, one current and one voltage waveform.)
- Current phasors are normally shown with closed arrowheads, and voltages are normally shown with open arrowheads.

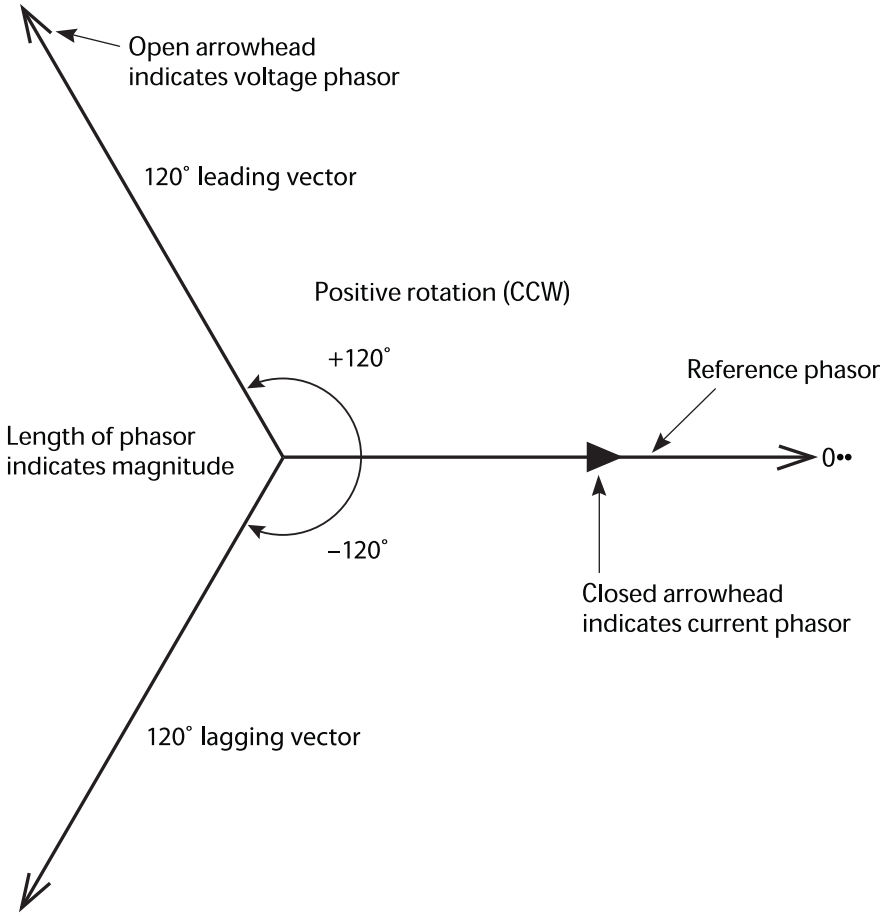


Figure 7—Phasor conventions

Subscript notation

Phasors are often labelled with a double subscript. The order of the two subscripts is very important. Many people get confused when analyzing three-phase circuits because they do not fully understand the importance of this order.

The two subscripts indicate the two points between which a voltage exists. When double-subscript notation is applied to an AC circuit, the sequence of the subscripts indicates the direction in which the current, the emf and the potential difference are assumed to be positive.

Emf is assumed to be a voltage rise, and potential difference is assumed to be a voltage drop. A voltage rise occurs when the circuit receives energy from something outside the circuit. A voltage drop occurs when the circuit dissipates energy to something outside the circuit.

Source	Load
• *Voltage* across the source (a voltage rise) is considered to be positive when it goes from positive to negative.	• *Voltage* across a load (a voltage drop) is considered to be positive when it goes from a negative to a positive.
• That is, emf or a rise in potential takes place when an electron is moved from a more positive point to a more negative point.	• That is, a drop in potential takes place when an electron is moved from a more negative point to a more positive point.
• The first subscript is more positive.	• The first subscript is more negative.
• *Current* through a source is considered to be positive when it is in a direction that increases the potential energy of the electron.	• *Current* through a load is considered to be positive when it is in a direction that decreases the potential energy of the electron.
• Thus, current is considered to be positive through a source when it flows from a more positive point to a more negative point.	• Thus, current is considered to be positive through a load when it flows from a more negative point to a more positive point.
• The first subscript is more positive.	• The first subscript is more negative.

The first subscript always represents the end of the circuit component at which the current tracing-loop enters.

The second subscript always represents the end of the circuit component from which the current tracing loop leaves. See Figure 8.

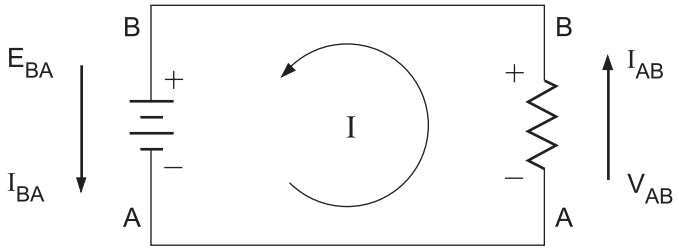


Figure 8—Current and voltage subscripts

Notice that this allows Kirchhoff's voltage law to be satisfied:

$$E_{BA} = V_{AB}$$
$$E_{BA} - V_{BA} = 0$$

Here the letter symbol E is used to represent an emf (or voltage rise) and the letter symbol V is used to represent a potential difference (or voltage drop).

Reversing the order of the subscripts means the same thing as changing the direction of the phasor by 180°.

Current tracing loops

By convention the current tracing loops are drawn to leave the source through the lines A, B and C, and to return to the source through the neutral. See Figure 9.

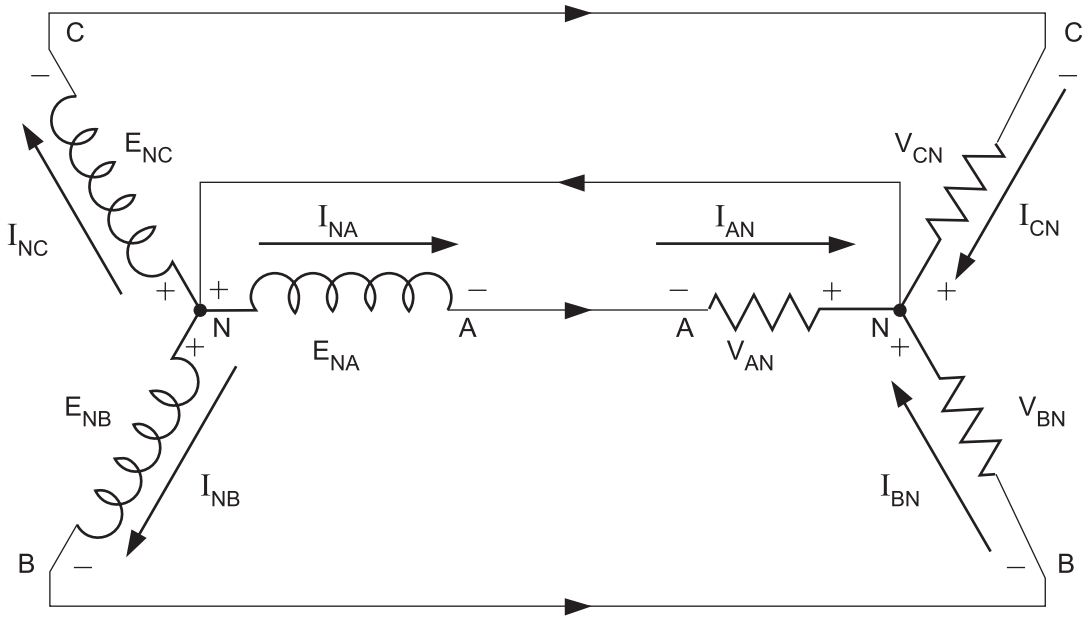


Figure 9—Current tracing loops

For this to take place, the outside ends of the source windings must be negative with respect to the star point ends. Figure 10 shows the voltage phasors and current phasors for this Y-connected source.

Notice that the voltage phasors are drawn with open arrowheads while the current phasors are drawn with closed arrowheads.

- Voltage phasor E_{NA} represents the voltage across the A winding. Because it is drawn at 3 o'clock on the page, it is the reference phasor. This means the sine wave representing the voltage E_{NA} at angle 0° is positive going from zero. (Positive going from zero means that it is at the start of its sine wave cycle where N is positive with respect to A.)
- Voltage phasor E_{NB} drawn at the angle of 240° (or –120°) means that it must be positive going from zero at 120° after E_{NA}.
- Similarly, E_{NC} drawn at the angle 120° means that it must be positive going from zero at 240° after E_{NA}.
- The current phasor I_{NA} represents the current flowing through the A winding. Since the load is purely resistive, the current through winding A is in phase with the voltage across winding A. Therefore, it is also drawn at 3 o'clock on the page. This means the sine wave representing the current I_{NA} is positive going from zero at angle 0°.
- Current phasor I_{NB} is positive going from zero at 120° after I_{NA}.
- I_{NC} is positive going from zero at 240° after I_{NA}.

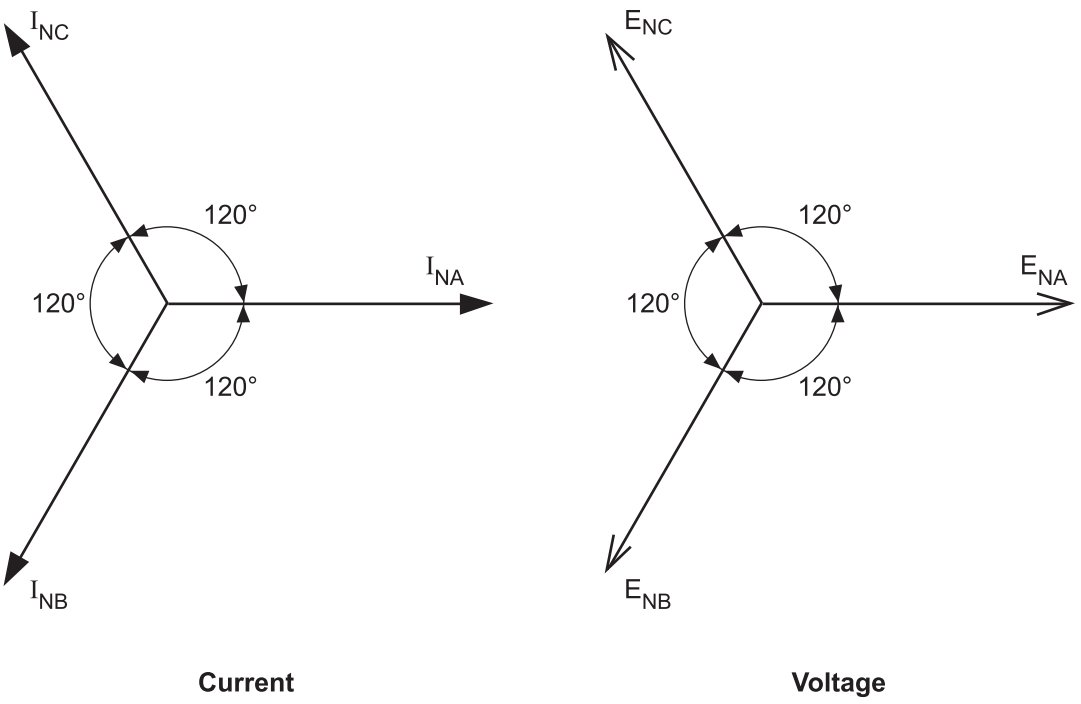


Figure 10—Voltage and current phasors

Now do Self-Test 1 and check your answers.

Self-Test 1

1. What does the term *polyphase* mean?

2. List four advantages of a three-phase system.

3. What is the phase angle between the three generated voltages of a three-phase alternator?

4. What are the two possible configurations for the leads of a three-phase alternator?

5. What does the term *phase sequence* mean?

6. How is the reversal of phase sequence normally accomplished?

_____

_____

_____

7. The CEC requires that the conductors of a three-phase system be identified with the following colours:

   a. Phase A _____

   b. Phase B _____

   c. Phase C _____

   d. Neutral _____ or _____

8. The length of the phasor is most often used to represent the _____ value of the sine wave.

9. The positive direction of phasor rotation is _____ .

10. Is it important that all the sine waves represented on the same phasor diagram have the same frequency?

_____

11. Voltage phasors are generally drawn with _____ arrowheads.

12. When subscript notation is used to represent the voltage across a source, the first subscript has _____ polarity with respect to the second.

13. When subscript notation is used to represent the voltage across a load, the first subscript has _____ polarity with respect to the second.

14. The first subscript always represents the end of the circuit component at which the current _____ the circuit component.

15. Reversing the order of the subscripts causes the direction of the phasor to change by _____ degrees.

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 2:

Describe the characteristics of the wye connection

Voltages in a Y-connected system

Figure 1 shows a three-phase, Y-connected source. The voltages across each of the armature windings are called *coil voltages*. The *magnitude* of each coil voltage shown in Figure 1 is 120 V. Because these voltages are generated across each phase of the alternator, they are also referred to as *phase voltages*.

Therefore the three coil voltages are:

Winding A: $E_{NA} = 120\ V \angle 0^o$

Winding B: $E_{NB} = 120\ V \angle 240^o$ or $120\ V \angle -120^o$

Winding C: $E_{NC} = 120\ V \angle 120^o$

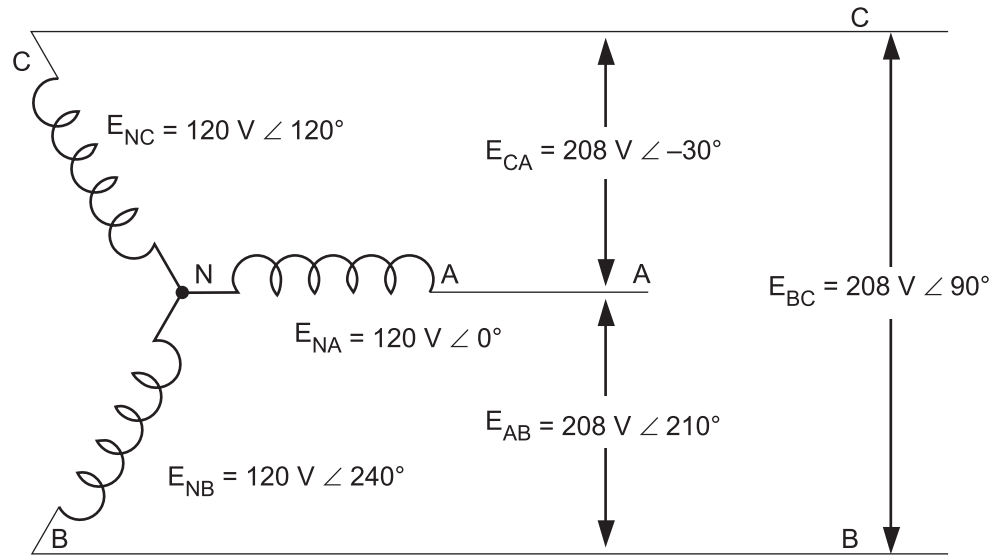


Figure 1—Y-connected source

The voltages between any two of the three line terminals A, B or C are referred to as the *line-to-line voltages* or simply as the *line voltages*. These voltages are equal to the phasor difference between the two corresponding phase voltages. Because the two coil voltages are not in phase with each other, they cannot be subtracted algebraically but must be dealt with as phasors.

We will calculate the three line voltages in this case to be:

$E_{AB} = 208\ V \angle 210^o$

$E_{BC} = 208\ V \angle 90^o$

$E_{CA} = 208\ V \angle -30^o$

Figure 2 shows two phasor representations of the six voltages. Figure 2(a) is the normal way to show phasors, all originating from a common point. In Figure 2(b) the angles and magnitudes of the phasors have not changed, but the line voltage phasors have been moved to show the relationship between line voltages and phase voltages more clearly.

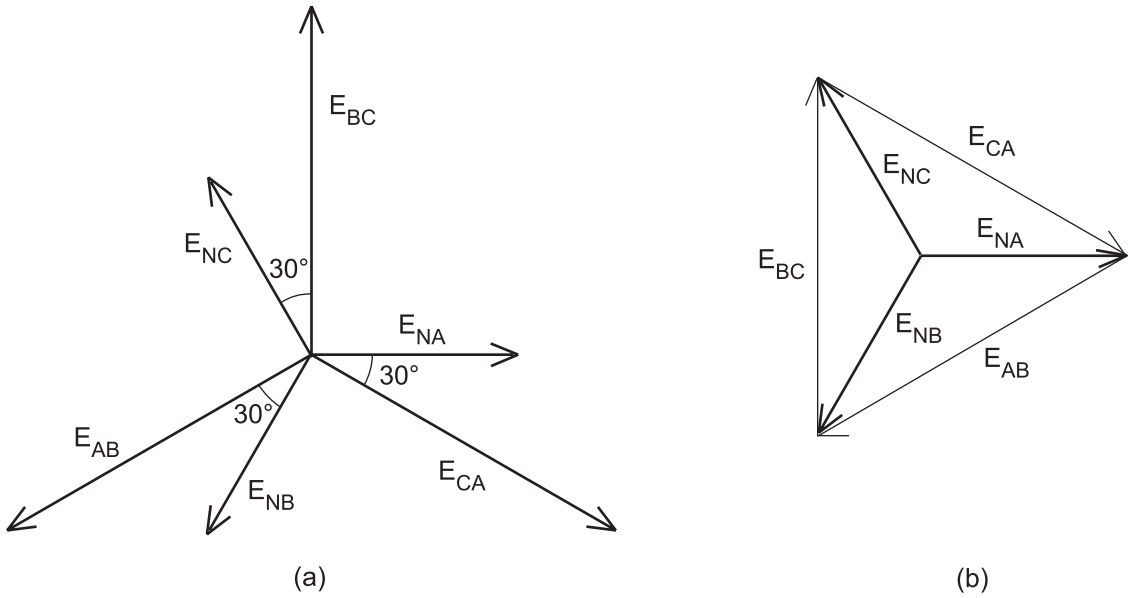


Figure 2—Voltages for a Y-connected generator

The voltage between any two points can be found by subtracting the voltage at one point (with respect to a common reference) from the voltage at the other point (with respect to the same reference).

The voltage rise from A to B (E_{AB}) can be found by:

$$E_{AB} = E_{NB} - E_{NA}$$

But, subtracting a phasor is the same as adding one in the opposite direction, so that:

$$E_{AB} = E_{NB} + E_{AN}$$

$$E_{AN} = 120\ \text{V} \angle 180^\circ$$

$$E_{AB} = 120\ \text{V} \angle 240^\circ + 120\ \text{V} \angle 180^\circ$$

$$= (120 \cos 240^\circ + \text{j}120 \sin 240^\circ) + (120 \cos 180^\circ + \text{j}120 \sin 180^\circ)$$

$$= (-60 - \text{j}103.9) + (-120 + \text{j}0)$$

$$= -180 + (-\text{j}103.9)$$

$$= \sqrt{180^2 + 103.9^2}\,\text{V} \angle \tan^{-1}\left(\frac{-103.9}{-180}\right)$$

$$= 208\ \text{V} \angle 210^\circ$$

On your calculator you may find that the $\tan^{-1}(-103.9/-180)$ is 30°, but you can see from the minus signs on the x and y coordinates that the angle must fall in quadrant 3. Therefore, the angle must be 180° + 30° = 210°.

The equation $E_{AB} = E_{NB} + E_{AN}$ can be seen graphically in Figure 2(b) by tracing:

- from A to B (E_{AB})
- from A to N (E_{AN})
- from N to B (E_{NB})

The same method can be used to solve for E_{BC} and E_{CA}:

$$E_{BC} = E_{NC} - E_{NB}$$

$$E_{BC} = E_{NC} + E_{BN}$$

$$E_{BN} = 120\ \text{V} \angle 60°$$

$$E_{BC} = 120\ \text{V} \angle 120° + 120\ \text{V} \angle 60°$$

$$= (120 \cos 120° + j120 \sin 120°) + (120 \cos 60° + j120 \sin 60°)$$

$$= (-60 + j103.9) + (60 + j103.9)$$

$$= 0 + (j208)$$

$$= \sqrt{0^2 + 208^2}\ \text{V} \angle \tan^{-1}\left(\frac{208}{0}\right)$$

$$= 208\ \text{V} \angle 90°$$

On your calculator you may find that you get an error when you try to divide 208 by zero. But you know that if the x-coordinate is 0 the opposite side equals the hypotenuse, and this only happens at 90° and 270°. The y-coordinate is positive, so the angle must be 90°.

$$E_{CA} = E_{NA} - E_{NC}$$

$$E_{CA} = E_{NA} + E_{CN}$$

$$E_{CN} = 120\ \text{V} \angle 300°$$

$$E_{CA} = 120\ \text{V} \angle 0° + 120\ \text{V} \angle 300°$$

$$= (120 \cos 0° + j120 \sin 0°) + (120 \cos 300° + j120 \sin 300°)$$

$$= (120 + j0) + (60 - j103.9)$$

$$= 180 + (-j103.9)$$

$$= \sqrt{180^2 + 103.9^2}\ \text{V} \angle \tan^{-1}\left(\frac{-103.9}{180}\right)$$

$$= 208\ \text{V} \angle -30°$$

The magnitude of the line voltage in a balanced Y-connected system is always 1.73 or $\sqrt{3}$ times the magnitude of the phase voltage. Let's look at how this number is arrived at:

Figure 3 is a 30° right-angle triangle and Figure 4 shows the relationship between E_{NA}, E_{NB} and E_{AB}.

$\cos\theta = \frac{\sqrt{3}}{2}$ = adjacent side over the hypotenuse

Bisect the angle between the two phase voltages in Figure 4 to create two 30° right-angle triangles of equal size.

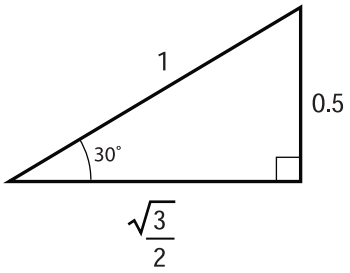


Figure 3—30° right-angle triangle

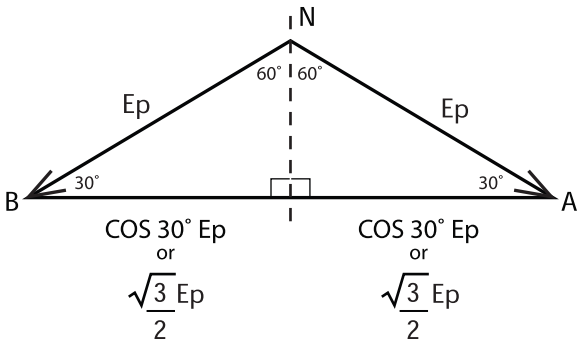


Figure 4—Line and phase voltages

Notice that the phasor E_{AB} is equal to the sum of the two sides adjacent to the 30° angles. Therefore, the magnitude of E_{AB} must equal:

$$E_{AB} = 2 \times \cos 30^{\circ} \times E_{phase}$$

$$= 2 \times \frac{\sqrt{3}}{2} \times E_{phase}$$

$$= \sqrt{3} \times E_{phase}$$

or

$$E_{line} = \sqrt{3} \times E_{phase} = 1.73 \times E_{ph}$$

Currents in a Y-connected system

Since each coil of the alternator is connected in series with one of the three lines, the current in each line must be the same as the current in each corresponding phase.

Therefore, the coil current, phase current and line current are all equal in a Y-connected, three-phase alternator. See Figure 5.

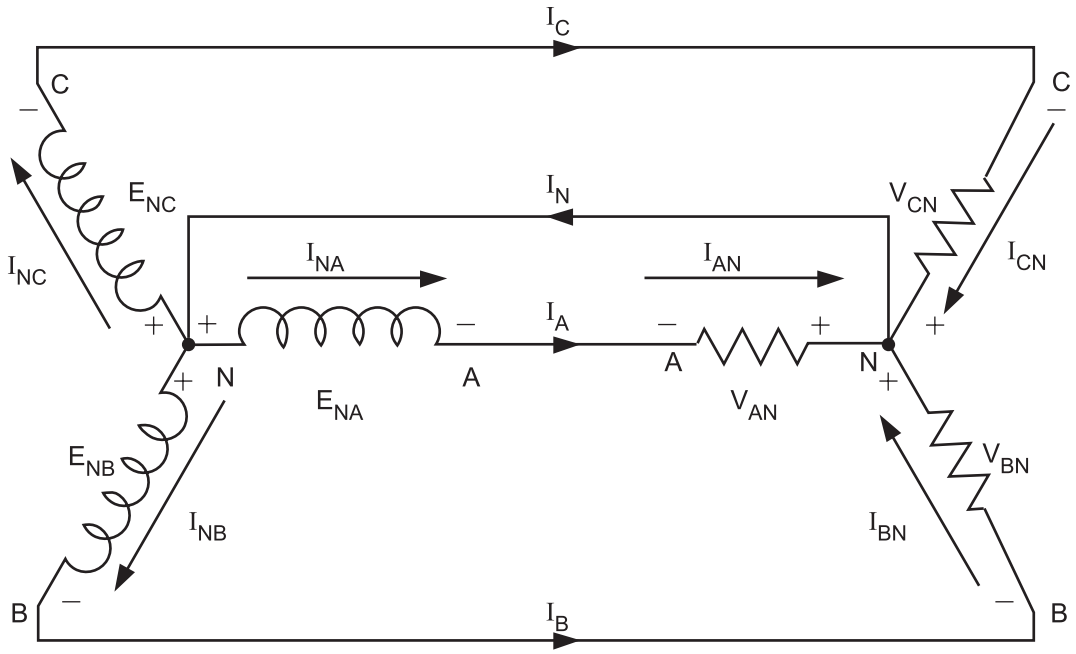


Figure 5—Current relationships

Figure 6 shows the relationship between the line voltages, phase voltages and line currents in a balanced, purely resistive, three-phase, Y-connected system.

Notice that the line current and the line voltage are out of phase with each other by 30°.

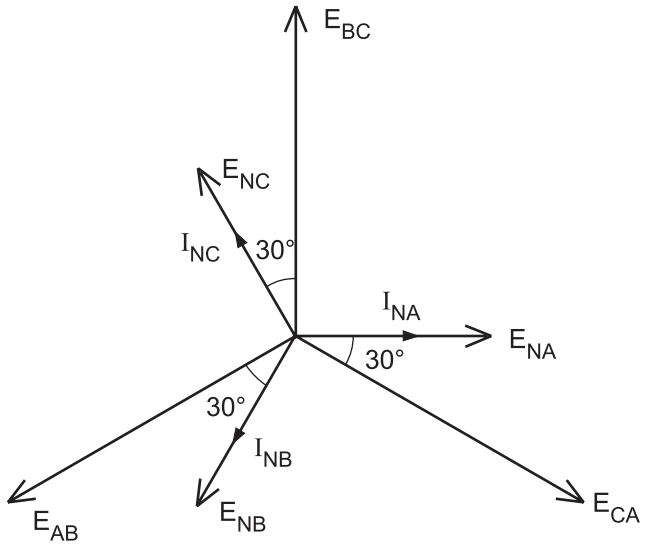


Figure 6—Voltage and current relationships

Notice that each of the Y-connected loads is connected directly across the phase voltage of the source. To satisfy Kirchhoff's voltage law for each of these loops, the voltage rise in each phase of the source must equal the voltage drop in each corresponding phase of the load.

$$E_{NA} = V_{AN}$$

$$E_{NB} = V_{BN}$$

$$E_{NC} = V_{CN}$$

Current in each phase of the load must equal the phase voltage divided by the phase impedance.

$$I_{AN} = \frac{V_{AN}}{Z_{AN}}$$

$$I_{BN} = \frac{V_{BN}}{Z_{BN}}$$

$$I_{CN} = \frac{V_{CN}}{Z_{CN}}$$

Because the source and the load are connected in series, the current in each phase of the source must equal the current in each phase of the load.

Reasons for grounding the star point

Except as specially provided for, the Canadian Electrical Code requires that all alternating current systems with a neutral be grounded. The CEC also specifies which conductor in the system must be connected to ground.

The conductor to be grounded in a three-phase, four-wire system is the conductor common to all three phases—that is, the neutral.

There are several reasons for grounding the neutral of a three-phase, four-wire system. They are:

- Grounding the neutral reduces the magnitude of transient overvoltages caused by such things as switching inductive loads and arcing faults. This is because the ground connection allows the quick drain of trapped charge on the system.
- Lightning protection is improved because line-to-ground lightning arresters with a lower voltage rating may be used. This is because, in the case of a line-to-ground fault on a grounded system, the maximum line-to-ground voltage is the phase voltage. On an ungrounded system it is the line voltage.
- Ground faults may be more easily located and eliminated. Protective devices will operate on the occurrence of the first fault when a grounded neutral is used. On an ungrounded system, two faults, each on different lines, are normally required before protective devices will operate.

- More effective system and equipment fault protection may be used. The low impedance ground path allows the fault current to rise to a value high enough for protective equipment to respond.
- The reduced magnitude of transient overvoltages reduces the stress on electrical insulation and extends the life of the electrical equipment. This decreases service costs and improves system reliability.
- When a line-to-ground fault occurs, maximum voltage-to-ground is limited to phase voltage in a grounded system. In an ungrounded system it may be as high as line voltage. Limiting the line-to-ground potential protects personnel and equipment.



Now do Self-Test 2 and check your answers.

Self-Test 2

1. If the magnitudes of the phase voltages of a three-phase, four-wire, wye-connected system are 300 volts, what are magnitudes of the line voltages?

2. If the magnitudes of the line voltages of a three-phase, four-wire, Y-connected system are 400 volts, what are the magnitudes of the phase voltages?

3. In a purely resistive, balanced Y-connected system, what is the magnitude of the phase angles between the line voltages and the line currents?

4. What is the mathematical relationship between the line currents and the phase currents in a Y-connected system?

5. What is the mathematical relationship between the line voltages and the phase voltages in a Y-connected system?

6. List four reasons for grounding the neutral in a three-phase, four-wire, Y-connected system.

7. Solve the following phasor-addition problem:
   $277\ V \angle 120^\circ + 277\ V \angle 60^\circ$

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 3:

Describe the characteristics of the delta connection

Voltages in a delta-connected system

Figure 1 shows a three-phase, delta-connected source. The magnitude of the voltage across each winding of this source is 120 VAC.

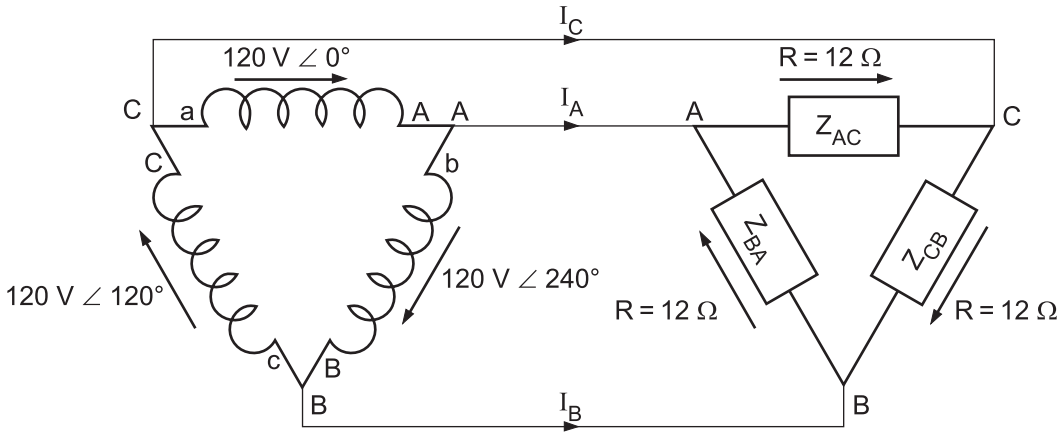


Figure 1—Delta-connected system

Therefore, the three coil voltages are:

Winding A: $E_{CA} = 120\ V \angle 0°$

Winding B: $E_{AB} = 120\ V \angle 240°$ or $120\ V \angle -120°$

Winding C: $E_{BC} = 120\ V \angle 120°$

Notice that each coil is connected directly across two line conductors. It follows that, in a Δ-connected system, the line voltage is equal to the phase voltage.

Currents in a delta-connected system

Since load impedance determines the current that will flow in a circuit, you must solve for currents at the load end of the circuit. The system in Figure 1 contains three balanced, purely resistive loads.

The voltage rise from C to A in the source (E_{CA}) must equal the voltage drop from A to C in the load (V_{AC}) in order to satisfy Kirchhoff's voltage law.

$$E_{CA} = V_{AC}$$

likewise $E_{AB} = V_{BA}$

and $E_{BC} = V_{CB}$

The currents in each of the phase loads must equal:

$$I_{AC} = \frac{V_{AC}}{Z_{AC}} = \frac{120\ V\ \angle\ 0^o}{12\ \Omega} = 10\ A\ \angle\ 0^o$$

$$I_{CB} = \frac{V_{CB}}{Z_{CB}} = \frac{120\ V\ \angle\ 120^o}{12\ \Omega} = 10\ A\ \angle\ 120^o$$

$$I_{BA} = \frac{V_{BA}}{Z_{BA}} = \frac{120\ V\ \angle\ 240^o}{12\ \Omega} = 10\ A\ \angle\ 240^o$$

Since all of the loads in Figure 1 are balanced, purely resistive loads, the currents in each phase are equal to each other and in phase with the phase voltages.

Notice that each junction consists of two phase windings and one line conductor. Kirchhoff's current law states that the phasor sum of currents leaving any junction in an AC circuit must equal the phasor sum of currents entering that junction.

Look at junction A at the load end of the system. Two currents enter the junction and one leaves it. Therefore, the equation for Kirchhoff's current law is:

$$I_{AC} = I_A + I_{BA}$$

Solving for the line current I_A, the equation becomes:

$$\begin{aligned} I_A &= I_{AC} - I_{BA} \\ &= I_{AC} + I_{AB} \\ &= 10\ A\ \angle 0^\circ + 10\ A\ \angle 60^\circ \\ &= (10 + j0) + (5 + j8.66) \\ &= 15 + j8.66 \\ &= \sqrt{15^2 + 8.66^2}\angle \tan^{-1}\left(\frac{8.66}{15}\right) \\ &= 17.3\ \ A\ \angle 30^\circ \end{aligned}$$

Similarly:

$$I_B + I_{CB} = I_{BA}$$

$$I_B = I_{BA} - I_{CB}$$

$$= I_{BA} + I_{BC}$$

$$= 10\ \text{A}\ \angle 240^\circ + 10\ \text{A}\ \angle -60^\circ$$

$$= (-5 - j8.66) + (5 - j8.66)$$

$$= 0 + (-j17.3)$$

$$= \sqrt{0^2 + 17.3^2}\ \angle \tan^{-1}\left(\frac{-17.3}{0}\right)$$

$$= 17.3\ \text{A}\ \angle -90^\circ$$

$$I_C + I_{AC} = I_{CB}$$

$$I_C = I_{CB} - I_{AC}$$

$$= I_{CB} + I_{CA}$$

$$= 10\ \text{A}\ \angle 120^\circ + 10\ \text{A}\ \angle -180^\circ$$

$$= (-5 + j8.66) + (-10 + j0)$$

$$= -15 + j8.66$$

$$= \sqrt{15^2 + 8.66^2}\ \angle \tan^{-1}\left(\frac{8.66}{-15}\right)$$

$$= 17.3\ \text{A}\ \angle -150^\circ$$

Figure 2 shows the relationship between line currents and phase currents for a balanced, purely resistive, Δ-connected system.

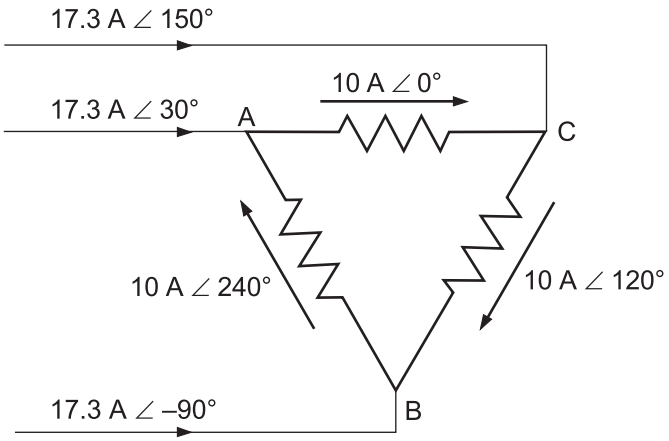


Figure 2—Currents for a Δ-connected system

In a Δ-connected system, the current is still assumed to travel from the source to the load in lines A, B and C. Since there is no neutral conductor, the sum of the three line currents must equal zero.

To illustrate this, use the line currents just calculated:

$$I_A + I_B + I_C = 0$$

$$\text{Total line current} = 17.3\text{ A} \angle 30° + 17.3\text{ A} \angle -90° + 17.3\text{ A} \angle 150°$$

$$= (15 + j8.66) + (0 - j17.3) + (-15 + j8.66)$$

$$= 0$$

Solving for the currents generated in the phase windings of the delta source involves the use of phasor algebra combined with simultaneous equations. For this reason it will not be covered.

The magnitude of the line currents in a balanced Δ-connected system is always $\sqrt{3}$ or 1.73 times the magnitude of the phase currents. This number is arrived at the same way as was used for line voltages in a balanced Y-connected system.

Remember that for the currents at point A in Figure 2, $I_A = I_{AC} + I_{AB}$. Figure 3 shows more about the relationship between I_{AC}, I_{AB} and I_A.

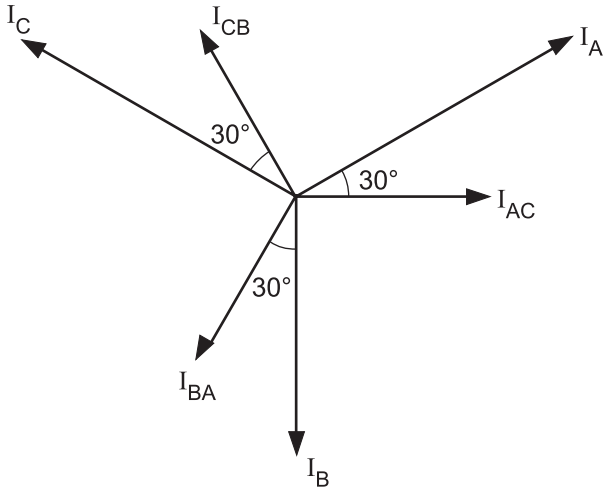


Figure 3—Line and phase currents

If the angle between the two phase currents is bisected it, creates two 30° right-angle triangles of equal size. Notice that the phasor I_A is equal to the sum of the two sides adjacent to the 30° angles. Therefore:

$$I_A = 2 \times \cos 30° \times I_{phase}$$

$$= 2 \times \frac{\sqrt{3}}{2} \times I_{phase}$$

$$= \sqrt{3} \times I_{phase}$$

$$I_{line} = \sqrt{3} \times I_{phase}$$

Ungrounded systems and ground-fault detection

Most of the time three-phase, three-wire delta systems are ungrounded systems. Ideally, there is no potential difference between any one of the secondary phase conductors and ground in an ungrounded system. Therefore, if one phase is accidentally grounded, none of the overcurrent devices are activated. However, a short circuit results if another phase develops a ground fault.

The CEC requires that, to indicate the presence of a ground fault, a ground detection device be installed on a wiring system supplied by an ungrounded supply. Figure 4 illustrates the use of simple indicating lights to detect the presence of a ground fault.

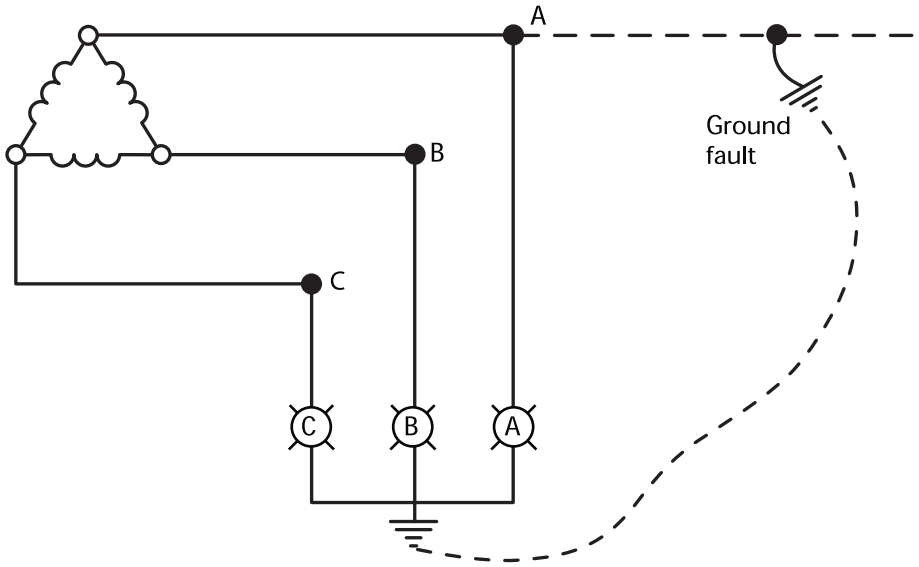


Figure 4—Ground fault indicating lights in a three-phase, ungrounded system

These indicating lights are wye-connected to the ungrounded supply, and the wye-point is purposely grounded. Each of the lamps has a voltage rating equal to the line voltage of the system. They function as follows:

- Under normal conditions, with no ground fault on the system, all three lights glow dimly with approximately 58% of the line voltage across each lamp.
- If a ground fault occurs on one of the three phase lines, then the light connected to the faulty line becomes dark because the grounding path shunts the lamp current. The remaining two lamps glow brightly, since they are now connected line-to-line through the grounding paths.

Note that a ground-detection device does not eliminate a ground fault from a circuit (such as tripping of branch-overcurrent devices). Nor does it identify its location. It simply indicates the presence of the fault on a particular line within the system.

Three-phase, four-wire delta system

Figure 5 shows a three-phase, four-wire, Δ-connected system providing a grounded 120 V/240 V, single-phase supply for lighting, as well as three-phase loads. Figure 6 is a phasor representation of the four-wire Δ source in Figure 5.

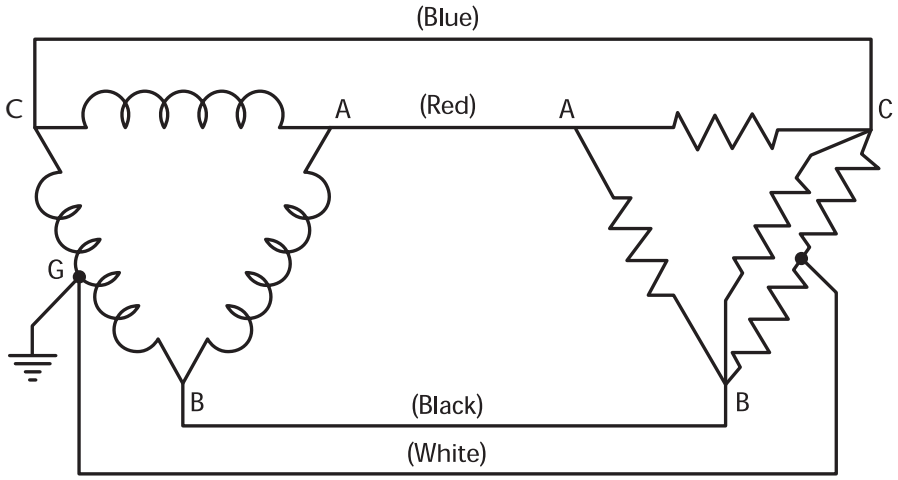


Figure 5—Three-phase, four-wire Δ-connected system

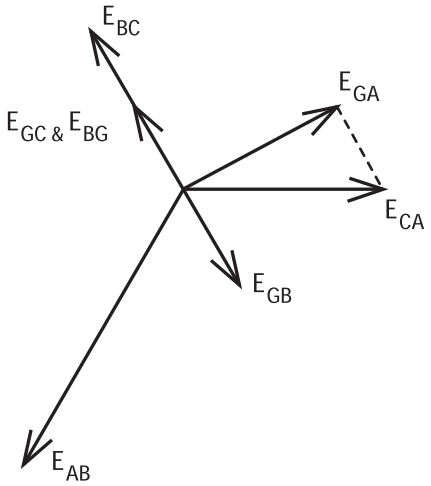


Figure 6—Phasor representation of a four-wire Δ source

Section 4 of the CEC requires that the four conductors be colour-marked:

- Phase A = Red
- Phase B = Black
- Phase C = Blue
- Neutral = White or natural grey

The code also requires that the phase A conductor have the higher voltage to ground. Notice that the four-wire Δ system is grounded in the middle of one of the phase windings. This means that voltages from the three lines with respect to ground are not equal.

Knowing that each phase voltage is 240 V and that the one winding is centre-tapped, you can calculate the voltages between the lines and ground.

The centre-tapping effectively creates four winding voltages:

$E_{GC} = 120\ V \angle\ 120°$

$E_{BG} = 120\ V \angle\ 120°$

$E_{AB} = 240\ V \angle\ 240°$

$E_{CA} = 240\ V \angle\ 0°$

Knowing these four voltages, you can calculate the three voltages that exist between the line conductors and ground:

$$E_{GC} = 120\ \ V\angle 120°$$

$$E_{GB} = E_{BG} = 120\ \ V \angle -60°$$

$$E_{GA} = E_{GC} + E_{CA}$$

$$= 120\ \ V \angle 120° + 240\ \ V \angle 0°$$

$$= (-60 + j103.9) + (240 + j0)$$

$$= 180 + j103.9$$

$$= \sqrt{180^2 + 103.9^2}\,V\angle \tan^{-1}\left(\frac{103.9}{180}\right)$$

$$= 208\ \ V\angle 30°$$

Because the voltage on the A line with respect to ground is higher than the other two lines, the A line is referred to as the *high leg*.

The CEC requires that:

- The grounded conductor be located in a compartment provided for single-phase connections, and
- The phase conductor with the higher voltage to ground be suitably separated from that compartment.

This also means that the grounded conductor must not be in a panel together with the three line conductors. It would be possible for someone to mistake this for a four-wire, Y-connected supply.

Now do Self-Test 3 and check your answers.

Self-Test 3

1. If the magnitudes of the phase voltages in a three-phase, three-wire, delta-connected system are 240 V, what are the magnitudes of the line voltages?

2. If the magnitudes of the phase currents in a three-phase, three-wire, balanced Δ system are 200 amperes, what are the magnitudes of the line currents?

3. In a purely resistive, balanced, Δ-connected system, what is the magnitude of the phase angles between the line voltages and the line currents?

4. What is the mathematical relationship between the line currents and the phase currents in a Δ-connected system?

5. What is the mathematical relationship between the line voltages and the phase voltages in a Δ-connected system?

6. Are three-wire, Δ systems normally grounded or ungrounded?

7. In a three-phase, four-wire, grounded Δ system, which phase conductor is required to be the one with the higher voltage to ground?

8. Is it permissible to put the grounded conductor and the high leg of a three-phase, four-wire, Δ system in the same panel compartment?

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 4:

Calculate voltage, current and power for balanced three-phase circuits

To solve problems in three-phase circuits, you must apply the following equations:

For wye connections:

$$E_L = \sqrt{3} \times E_P \text{ or } E_P = \frac{E_L}{\sqrt{3}} \text{ and } I_L = I_P$$

For delta connections:

$$I_L = \sqrt{3} \times I_P \text{ or } I_P = \frac{I_L}{\sqrt{3}} \text{ and } E_L = E_P$$

For both connections:

$$VA_T = \sqrt{3} \times E_L \times I_L \text{ or } VA_T = 3 \times E_P \times I_P$$

$$P_T = \sqrt{3} \times E_L \times I_L \times PF \text{ or } P_T = 3 \times E_P \times I_P \times PF$$

where PF = cos θ and θ is the angle between phase voltage and phase current.

To calculate values in three-phase circuits, normally start at the load end. Based on the type of connection and line voltage available, calculate the load current and power values per phase, and translate them back into line or three-phase values. To simplify, you may look upon three-phase loads as three single-phase loads that can be connected in either a delta or wye (star) configuration.

Balanced loads

Most three-phase loads are *balanced* loads. That is, each phase has the same values of impedance and power factor, and has the same value of three-phase voltage impressed across it. Unbalanced loading normally occurs under a fault condition, or when single-phase loads are connected in addition to three-phase loads.

In practice, if you measured the same value of current in each of the three-phase lines, you would usually consider the load as balanced. A clamp-on ammeter connected simultaneously around all three line conductors of a balanced load should read zero.

Sample calculations

Following are examples of three-phase calculations involving balanced circuits. There are four different source-load combinations:

- a wye-connected source with a wye-connected load
- a wye-connected source with a delta-connected load
- a delta-connected source with a delta-connected load
- a delta-connected source with a wye-connected load

Example 1: Each phase coil of a wye-connected alternator generates an output of 120 V. The load connected to this three-phase alternator consists of three 10 Ω impedances connected in wye as shown in Figure 1. Each impedance has a power factor of 80% lag. Determine the following:

a. terminal voltage output of the alternator

b. line current to the load bank

c. three-phase power output of the alternator

d. total kVA load on the alternator

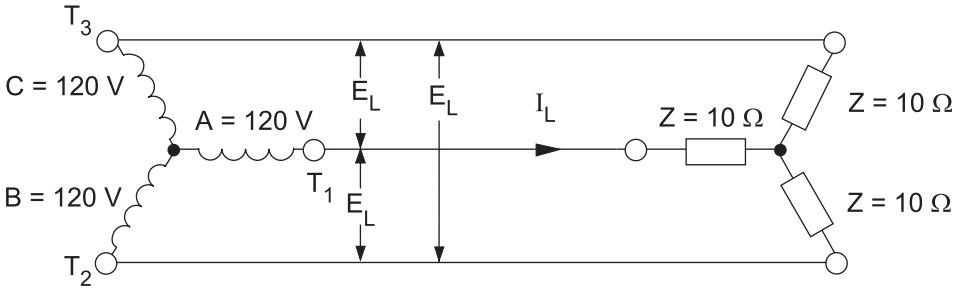


Figure 1—Wye-connected source with wye-connected load

Solutions:

a. The terminal voltage output is the line voltage. Since the alternator is connected in wye, the line voltage is:

$$E_L = \sqrt{3} \times E_P = 1.732 \times 120\ V = 208\ V$$

b. The line current is determined by the current per phase at the load. Since each phase of the load has an impedance of 10 Ω and the load is connected in wye, you can determine the phase current by applying Ohm's law.

Note that, although the line voltage is 208 V, the voltage across each impedance is:

$$E_P = \frac{E_L}{\sqrt{3}} = \frac{208\ V}{1.732} = 120\ V$$

The current through each impedance is:

$$I_P = \frac{E_p}{Z_P} = \frac{120\ V}{10\ \Omega} = 12\ A$$

For a wye connection, the line current is the same value as the phase current. Therefore:

$$I_P = I_L = 12\ A$$

c. Neglecting line losses, the power output of the alternator is the same as the power developed by the load impedances. To calculate power at the load, use either phase or line values of voltage and current.

Using phase values, the total power is:

$$P_T = 3 \times E_P \times I_p \times PF$$
$$= 3 \times 120\ V \times 12\ A \times 0.8$$
$$= 3456\ W$$

Using line values, the total power is:

$$P_T = \sqrt{3} \times E_L \times I_L \times PF$$
$$= \sqrt{3} \times 208\ V \times 12\ A \times 0.8$$
$$= 3459\ W$$

Note the slight discrepancy between answers because of the inaccuracy of using line-value figures that are *rounded off*. In this example, both calculated values of wattage may be rounded off to 3460.

d. Knowing the watts and power factor, the total kVA is calculated as:

$$VA = \frac{P_T}{PF} = \frac{3456}{0.8} = 4320\ VA \text{ or } 4.32\ kVA$$

Example 2: The three load impedances of Example 1 are reconnected in delta to the same alternator lines as shown in Figure 2. Under this condition, determine the following:

a. terminal voltage output of the alternator

b. line current to the load

c. three-phase power output of the alternator

d. total kVA load on the alternator

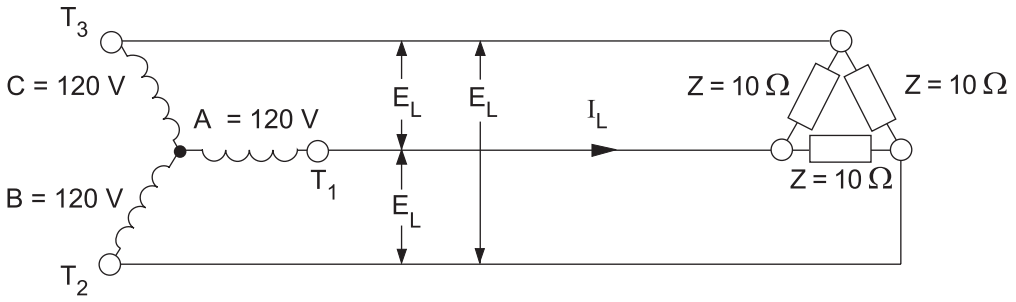


Figure 2—Wye-connected source with delta-connected load

Solutions:

a. As in the previous example, the terminal voltage output of the alternator is the same. Since each phase coil generates 120 V and the coils are connected in wye, the terminal voltage is still 208 V.

$$E_L = \sqrt{3} \times E_P = 1.732 \times 120 \text{ V} = 208 \text{ V}$$

b. The line current is determined by the current per phase at the load. Since the load is connected in delta, the line voltage is the same as the phase voltage. The current per phase is:

$$I_P = \frac{E_P}{Z_P} = \frac{208 \text{ V}}{10 \ \Omega} = 20.8 \text{ A}$$

For a delta connection, the line current is:

$$I_L = \sqrt{3} \times I_P = 1.732 \times 20.8 \text{ A} = 36 \text{ A}$$

c. Neglecting line losses, the power output of the alternator is the same as the power developed by the load impedances. To calculate power at the load, you can use either phase or line values of voltage and current. Using line values, the total power is calculated as:

$$P_T = \sqrt{3} \times E_L \times I_L \times PF$$

$$= 1.732 \times 208 \text{ V} \times 36 \text{ A} \times 0.8$$

$$= 10\,375 \text{ W} \approx 10.4 \text{ kW}$$

d. Knowing the watts and power factor, the total kVA is calculated as:

$$VA = \frac{P_T}{PF} = \frac{10.4 \text{ kW}}{0.8} = 13 \text{ kVA}$$

Note that, when the same load is reconnected in delta to the same supply, the power (and kVA) developed by the load bank is three times greater than when it is connected in wye.

Example 3: Each phase coil of a delta-connected alternator generates an output of 120 V. The load connected to this three-phase alternator consists of three 10 Ω resistors connected in delta as shown in Figure 3. Determine the following:

a. terminal voltage output of the alternator

b. line current to the load

c. three-phase power output of the alternator

d. total kVA load on the alternator

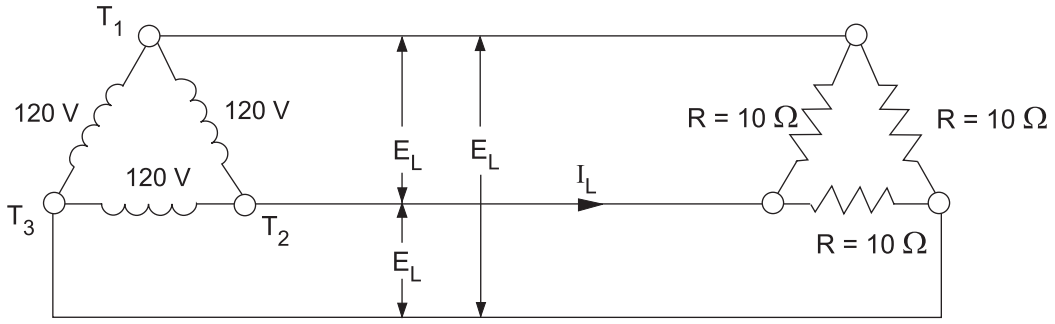


Figure 3—Delta-connected source with delta-connected load

Solutions:

a. Since the alternator is connected in delta, the coil voltage and the line voltage are the same:

$E_L = E_P = 120\ V$

b. Apply Ohm's law to calculate the current drawn by each resistance in the delta-connected load bank:

$$I_P = \frac{E_P}{R_P} = \frac{120\ V}{10\ \Omega} = 12\ A$$

The line current is:

$$I_L = \sqrt{3} \times I_P = 1.732 \times 12\ A = 20.8\ A$$

c. Neglecting line losses, the power output of the alternator is the same as the power developed by the load resistors. As before, you can use either phase or line values to calculate the total power. Using phase values, the power is:

$$P_T = 3 \times E_P \times I_P \times PF$$

$$= 3 \times 120\ V \times 12\ A \times 1$$

$$= 4320\ W \text{ or } 4.32\ kW$$

d. Since the load consists of resistors, the power factor is unity, and the true power is equal to the apparent power.

$kW = kVA = 4.32$

Example 4: The three load resistors of Example 3 are reconnected in wye to the same alternator lines as shown in Figure 4. Under this condition, determine the following:

a. terminal voltage output of the alternator

b. line current to the load bank

c. current in each phase coil of the alternator

d. three-phase power output of the alternator

e. total kVA load on the alternator

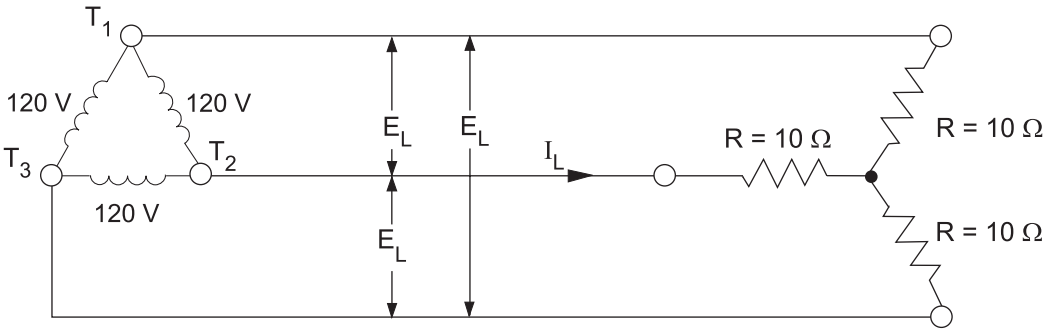


Figure 4—Delta-connected source with wye-connected load

Solutions:

a. As in the previous example, the terminal voltage output is the same as the coil voltage since the source is connected in delta.

$$E_L = E_P = 120 \text{ V}$$

b. Since the line voltage is 120 V, and the resistors are connected in wye, the voltage across each resistor is:

$$E_P = \frac{E_L}{\sqrt{3}} = \frac{120 \text{ V}}{1.732} = 69.3 \text{ V}$$

The current drawn by each resistor is:

$$I_P = \frac{E_P}{R_P} = \frac{69.3 \text{ V}}{10 \ \Omega} = 6.93 \text{ A}$$

The line current for a wye-connected load is the same value as the phase current. Therefore:

$$I_L = I_P = 6.93 \text{ A}$$

c. The coils of the source are connected in delta. Since the line current output from the alternator is 6.93 A, the phase current for each alternator coil is:

$$I_P = \frac{I_L}{\sqrt{3}} = \frac{6.93 \text{ A}}{1.732} = 4 \text{ A}$$

d. The total power output from the alternator can be calculated using either line values or phase values. Using line values:

$$P_T = \sqrt{3} \times E_L \times I_L \times PF$$

$$= 1.732 \times 120\ V \times 6.93\ A \times 1$$

$$= 1440\ W \approx 1.44\ kW$$

e. Since the load has unity PF, the kVA output is the same as the kW output of the alternator. Therefore:

kVA = kW = 1.44

The preceding examples illustrate the four different source-load combinations for three-phase circuits. For each case, the type of source connection (either wye or delta) determines the three-phase line voltage for the circuit. How the load is connected (either wye or delta) to this three-phase supply then determines the magnitude of line currents.

In solving three-phase circuit configurations, you will usually convert from phase-and-line values at the load to line-and-phase values at the source, and vice versa.

 Now do Self-Test 4 and check your answers.

Self-Test 4

1. Each phase of a wye-connected alternator delivers a full load current of 28.9 A at a coil voltage of 346 V, and a power factor of 80% lagging. Calculate the:

   a. voltage measured across the line terminals of the alternator

   _____

   b. three-phase power delivered to the load

   _____

   c. kVA rating of the alternator

   _____

   d. reactive power drawn by the load

   _____

2. For the circuit in Figure 1, calculate the:

   a. line voltage output from the alternator

   _____

   b. value of current through each load resistor

   _____

   c. current in each phase winding of the alternator

   _____

   d. kVA output of the alternator

   _____

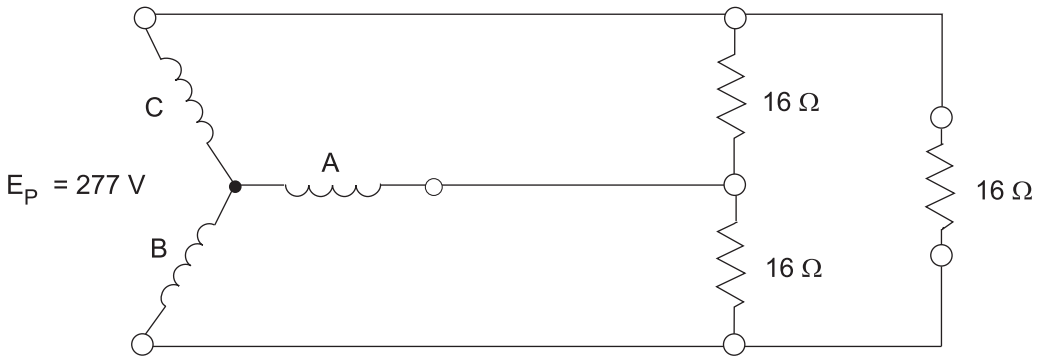


Figure 1—Wye-connected source and delta-connected load

3. Three resistance heaters are delta-connected to a 480 V, three-phase supply. If each resistor has a value of 12 Ω, calculate the:

   a. three-phase line current

   b. total power developed by the load

4. Three resistors of equal value are delta-connected to a 600 V, three-phase supply. If the total power used by the load is 54 kW, determine the:

   a. current through each resistor

   b. line current

   c. ohmic value of each resistor

5. Briefly explain what the term *balanced load*. What could cause a three-phase load to be unbalanced?

6. Three equal values of impedance are connected in wye to a 208 V, three-phase supply. Each impedance has a value of 12 Ω with a PF of 95% lagging. Determine the:

   a. phase current

b. line current

c. total power in kW

7. A wye-connected induction motor delivers 100 hp when operating on a 600 V, three-phase line at 90% lag PF. The motor draws a line current of 92 A. Determine the:

   a. voltage rating of each phase winding of the motor

   b. kW per phase used by the motor

8. Six 44 Ω resistors are connected so that three form a delta connection and three form a wye connection. The two banks of resistors are then connected in parallel to a 440 V, three-phase supply. Calculate the:

   a. current in each of the wye-connected resistors

   b. current in each of the delta-connected resistors

   c. total line current

9. Each phase coil of a three-phase alternator has an internal resistance of 2 Ω as measured by an ohmmeter. Calculate the ohmmeter-scale reading across the line terminals if the alternator coils were internally connected in:

   a. wye

   b. delta

10. Three elements of an electric furnace are connected in delta to a three-phase, 480 V supply. The impedance of each element is 10 Ω with unity power factor.

    a. What is the total kW load?

    _____

    b. What would be the total kW load if the elements were reconnected in wye?

    _____

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 5:

Determine the neutral current in wye-connected circuits

Using a neutral wire in wye-connected systems has two advantages:

- A three-phase, four-wire circuit can supply both three-phase motor loads and single-phase lighting loads between line and neutral conductors. The neutral conductor carries any current imbalance back to the source.
- The neutral is a convenient point for grounding the system, which improves safety.

Figure 1 shows a three-phase, four-wire lighting circuit with balanced loading. As illustrated in the diagram, the neutral current (I_N) is the vector sum of the three-phase currents. Since the three-phase currents are equal in magnitude and time-displaced by 120 electrical degrees, their vector sum is equal to zero.

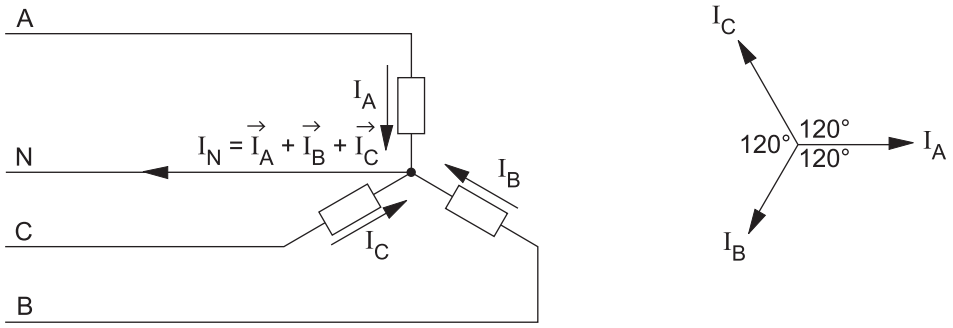


Figure 1—Balanced, three-phase, four-wire load

This is quickly verified by connecting the phase-current phasors head-to-tail, as shown in Figure 2. The phasors form a closed, equilateral triangle. This indicates that the three currents sum to zero. Therefore, the neutral current is zero.

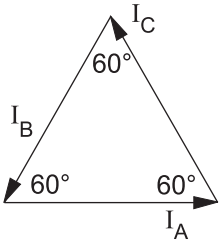


Figure 2—Phasor sum of equal currents

R If you have any difficulty understanding the following examples, you may wish to review Module 9, which deals with basic vector calculations.

Example 1: Three 10 Ω resistors are star-connected to a 120 V/208 V, three-phase, four-wire supply as shown in Figure 3. Calculate the current in:

a. each line conductor

b. the neutral conductor

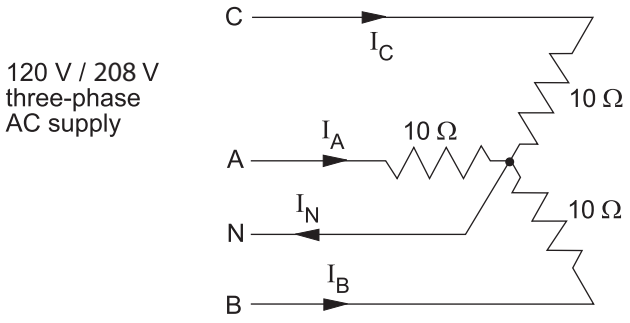


Figure 3—Balanced resistive loads

Solutions:

a. The line voltage in this three-phase system is 208 V, and since the load bank is connected in wye, the voltage across each resistor is 120 V.

$V_{AN} = 120\ V \angle\ 0°$

$V_{BN} = 120\ V \angle\ -120°$

$V_{CN} = 120\ V \angle\ +120°$

Applying Ohm's law, the current in each phase is:

$$I_A = \frac{V_{AN}}{Z_A} = \frac{120\ V \angle\ 0°}{10\ \Omega} = 12\ A \angle\ 0°$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{120\ V \angle\ -120°}{10\ \Omega} = 12\ A \angle -120°$$

$$I_C = \frac{V_{CN}}{Z_C} = \frac{120\ V \angle\ +120°}{10\ \Omega} = 12\ A \angle +120°$$

For a wye connection, since the phase current is equal to the line current, the phase values above are also the line values.

b. To determine the neutral current, you must add the three-phase currents vectorially using the formula:

$$\vec{I}_N = \vec{I}_A + \vec{I}_B + \vec{I}_C$$

$$= 12\ A \angle 0° + 12\ A \angle -120° + 12\ A \angle +120°$$

Converting to rectangular form:

$$I_N = (12 + j0) + (-6 - j10.4) + (-6 + j10.4)$$
$$= 0$$

When all three-phase (or line) currents are equal, there is no current in the neutral conductor. Normally, the neutral conductor carries current only when the load is unbalanced.

When wye-connected, four-wire systems are used to supply single-phase loads as well as three-phase loads, the total load is likely to become unbalanced, and the neutral conductor will carry current. The value of the neutral current can be determined by adding the current phasors.

Example 2: A 120 V/208 V, three-phase, four-wire system has the following load currents: line A = 12 A; line B = 10 A; line C = 6 A.

Calculate the neutral current.

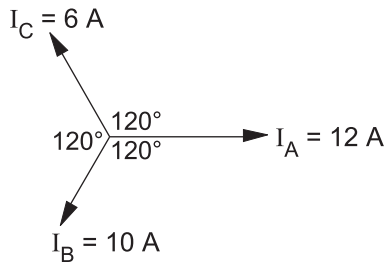


Figure 4—Phasor diagram for Example 2

Solution:
Assume the current in line A to be at 0°, the current in line B to be at –120° and the current in line C to be at 120°, as shown in Figure 4. Apply the vector formula:

$$\vec{I}_N = \vec{I}_A + \vec{I}_B + \vec{I}_C$$
$$= 12\ A \angle 0^o + 10\ A \angle -\ 120^o + 6\ A \angle +120^o$$
$$I_N = (12 + j0) + (-5 - j8.7) + (-3 + j5.2)$$
$$= 4 - j3.5$$

Converted back to polar form:

$$I_N = 5.3\ A \angle -40.9^o$$

The angle (–40.9°) associated with the vector quantity simply tells you where the waveform of the neutral current lies with respect to the other currents. It has no other significance for practical applications.

Another method of solving for the size of the neutral current of a three-phase, four-wire circuit uses an equation derived from the Law of Cosines:

$$n = \sqrt{a^2 + b^2 + c^2 - (ab) - (bc) - (ca)}$$

Using this equation with the values in the preceding example:

$$I_N = \sqrt{12^2 + 10^2 + 6^2 - (12 \times 10) - (10 \times 6) - (6 \times 12)}$$

$$= \sqrt{28} = 5.29 \approx 5.3 \text{ A}$$

Note that in three-phase circuits, a conductor is classified as a neutral when it is used as a common return path for all three line conductors (see Figure 5). When all three-phase conductors carry equal currents, the current in the neutral conductor is zero.

There may be instances when only two line conductors and the neutral are used to supply single-phase lighting loads (see Figure 5). In this case, the neutral is called the *common conductor*. When the two phase conductors carry equal currents, the current in the common conductor is the same magnitude as the line current. This is referred to in the Canadian Electrical Code.

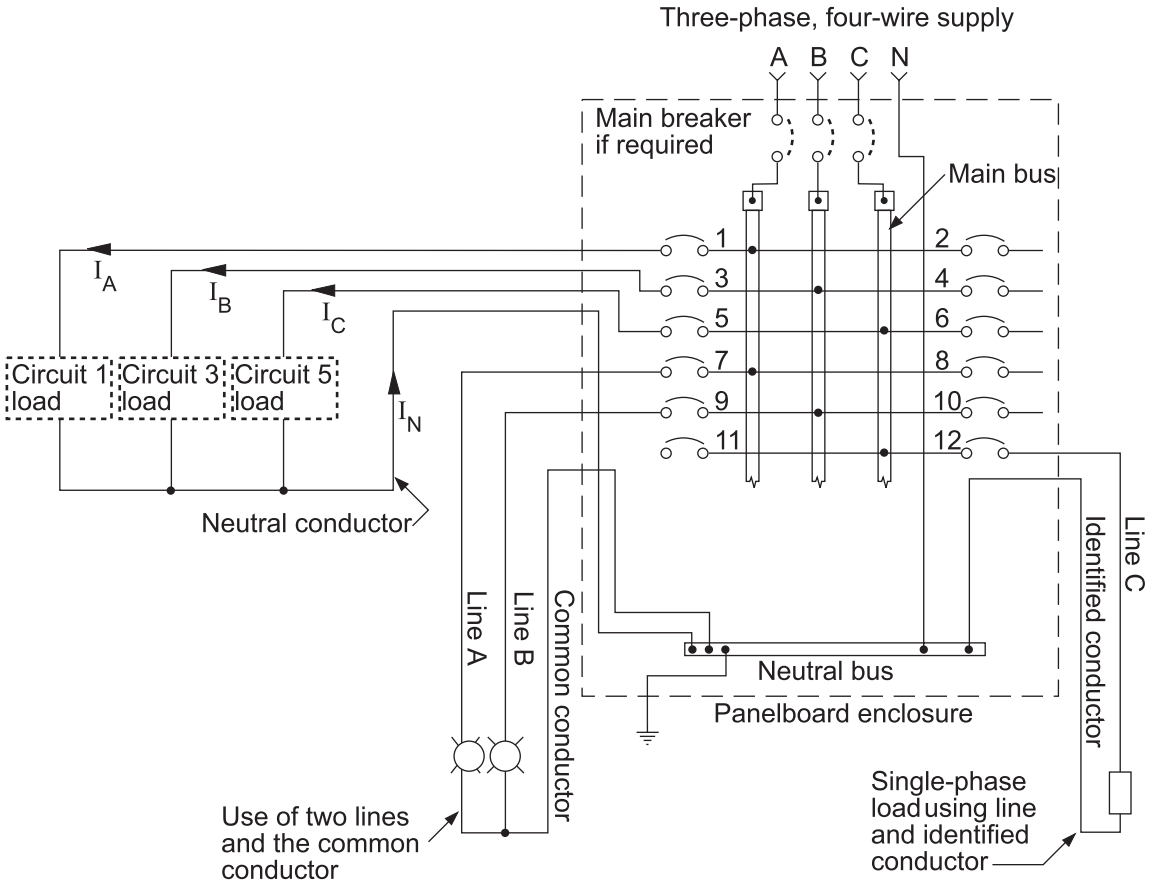


Figure 5—Neutral-conductor applications

Example 3: Two line conductors and the neutral of a 120 V/208 V, three-phase supply are used to supply some single-phase incandescent lighting. If the lighting is distributed with 2.5 kW connected from Line A to neutral, and also 2.5 kW connected from Line B to neutral, what is the current in the common (neutral) conductor?

Solution:
Calculate the magnitude of the current in each of the line conductors:

$$I_A = I_B = \frac{2500\ W}{120\ V} = 20.83\ A$$

Since the line currents are out of phase by 120°, add them vectorially to find the common current:

$$I_{com} = \vec{I}_A + \vec{I}_B$$

$$= 20.83\ A\ \angle\ 0° + 20.83\ A\ \angle\ -120°$$

Convert this to rectangular form:

$$I_{com} = (20.83 + j0) + (-10.42 - j18.04)$$

$$= 10.41 - j18.04$$

Convert back to polar form:

$$I_{com} = 20.83\ \angle\ -60°$$

Even though the circuit appears to be balanced, the common conductor carries the same value of current as each of the two line conductors. The current phasors for this circuit are shown in Figure 6.

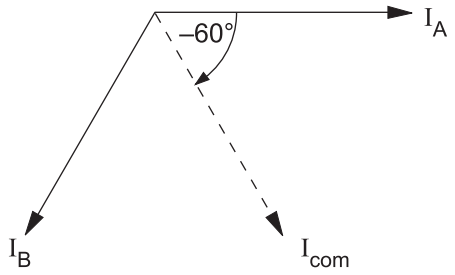


Figure 6—Current phasors for Example 3

Now do Self-Test 5 and check your answers.

Self-Test 5

1. Given the following values of line current in a three-phase, four-wire, wye-connected system, solve for the neutral current (IN). Consider all loads to be resistive.

   a. $I_A = I_B = 125$ A, and $I_C = 175$ A

   b. $I_A = 200$ A, $I_B = 225$ A, and $I_C = 275$ A

   c. $I_A = 20$ A, $I_B = 35$ A, and $I_C = 26$ A

2. For the resistive circuit shown in Figure 1, solve for the line and neutral currents.

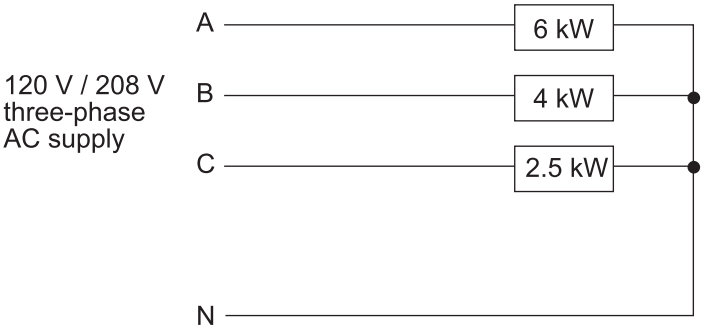


Figure 1—Circuit for question 2

3. Each phase of a wye-connected load has a power factor of 86%, with individual phase impedances of 3, 4 and 5 ohms. Determine the line and neutral currents when it is connected to a 120 V/208 V supply.

4. Two lines and the neutral of a three-phase, four-wire system are used to supply two 120 V incandescent-lighting loads. If the current in each of the two line wires is 10 A, what is the current in the common circuit (neutral) conductor?

5. A 120 V/208 V, four-wire distribution panel supplies incandescent-lighting and resistive-heating loads as follows:

   - Line A to neutral = 1200 W
   - Line B to neutral = 1800 W
   - Line C to neutral = 2400 W

   a. What is the current in the neutral?

   b. If the fuse in Line C were to blow, what would be the current in the neutral?

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 6:

Describe the effects of an open on wye and delta circuits

Most three-phase loads (such as motors) are considered as balanced loads. These loads can, however, become unbalanced under certain fault conditions such as *opens*. The opens can occur in the line or in the load.

Wye circuits

An open in the line

If an open occurs in one of the line conductors supplying the three-phase load, a condition referred to as *single-phasing* results. A typical example of this occurs if one of the line fuses blows open (see Figure 1).

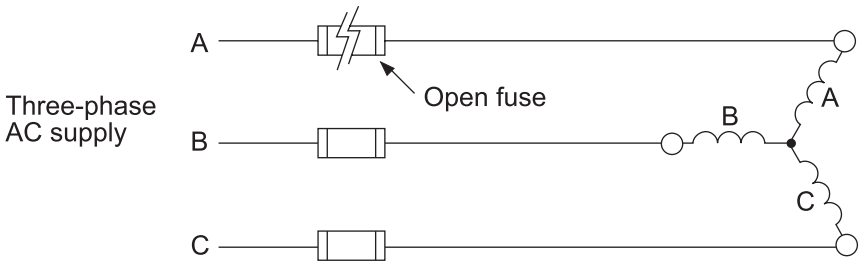


Figure 1—Single-phasing resulting from an open line

In this condition, the impedances of two phases are connected in series across one of the three-phase line combinations. In Figure 1, Lines B to C form one single-phase voltage active to the load, while lines A to B and Lines A to C are inactive (not carrying current) due to the open. In other words, only one of the three-phase voltages, or a single phase, is supplying current to the load.

Example 1: A three-phase resistance furnace consists of three 10 Ω resistors connected in wye supplied from a 208 V, three-wire supply. If one of the line fuses feeding this load blows open as shown in Figure 2, what is the:

a. current in each of the line conductors?

b. total power developed?

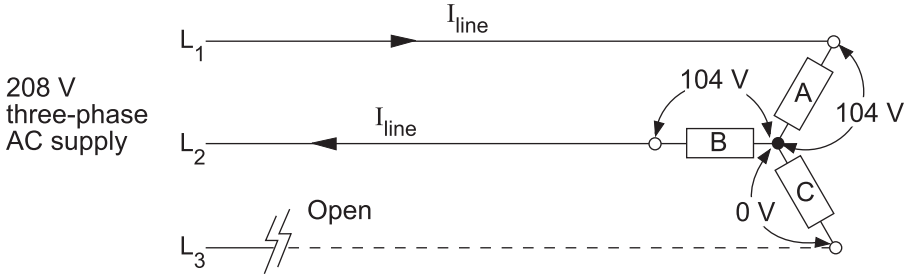


Figure 2—Two resistors in series as a result of an open

Solutions:

a. As shown by Figure 2, if line 3 becomes open, the resistors R_A and R_B are connected in series to a line-to-line voltage of 208 V.

Total resistance is:

$$R_T = R_A + R_B = 20$$

Total line current is:

$$I_L = \frac{E}{R} = \frac{208\ V}{20\ \Omega} = 10.4\ A$$

Since this is considered a single-phase circuit, the current flowing in line 1 is the same as the current in line 2. Since line 3 is open, there is no current in it.

Notice that since the two equal, 10 Ω resistors are connected in series across 208 V, the voltage drop across each resistor is 104 V (Ohm's law).

b. There are several different ways to solve for the circuit power. One way is to determine the power developed by each resistor, and simply add the two together for the total power.

For each resistor, power developed is:

$$P = E \times I \times PF$$

$$= 104\ V \times 10.4\ A \times 1 = 1081.6\ W$$

Total power is:

$$P_T = P_A + P_B$$

$$= 1081.6\ W + 1081.6\ W = 2163.2\ W$$

In this example, we calculated circuit values under an open-line-fault condition known as single-phasing. Compare them with the circuit values under normal, three-phase operating conditions. For a normal wye connection:

Line current is the same as the phase current:

$$I_L = \frac{E_P}{R_P} = \frac{120\ V}{10\ \Omega} = 12\ A$$

Total three-phase power is:

$$P_T = 3 \times P_P$$

$$= 3 \times E_P \times I_P \times PF$$

$$= 3 \times 120\ V \times 12\ A \times 1 = 4320\ W$$

Notice that, under a single-phase fault condition, the power developed by this three-phase load drops to approximately 50% (one-half) of the power developed under normal operating conditions.

Open in line with neutral connected to the load

Figure 3 shows the load of Example 1 connected to a 120 V/208 V, three-phase supply with an open in one of the lines, but this time the neutral is connected to the load. Compare this to the original load situation shown in Figure 2.

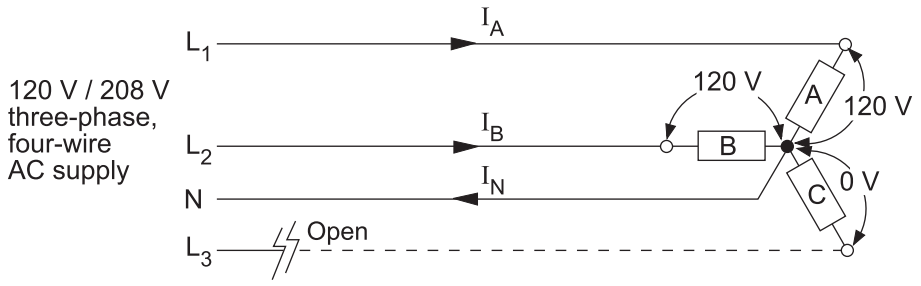


Figure 3—An open line in a four-wire circuit

When the neutral is connected and a fuse blows, current flow to one of the phases is interrupted. The remaining two phases of a resistive load operate the same as a normal three-phase load. The currents are:

$I_1 = I_2 = 12$ A and $I_3 = 0$

Recognize that the current in the neutral is also 12 amperes.

If an open in one of the lines occurs on a balanced, wye-connected load with the neutral connected, then the power developed is 66.7% (two-thirds) of normal. Notice that this is different from single-phasing.

An open in the load

If an open occurs within one of the load phases, as shown in Figure 4, the effects are identical to those described for single-phasing:

- The two good phases remain in series across one of the line voltages.
- No current flows through the faulty open phase.

Note that in this condition, a voltage may be measured across the open phase. Compare Figures 2 (previous) and 4.

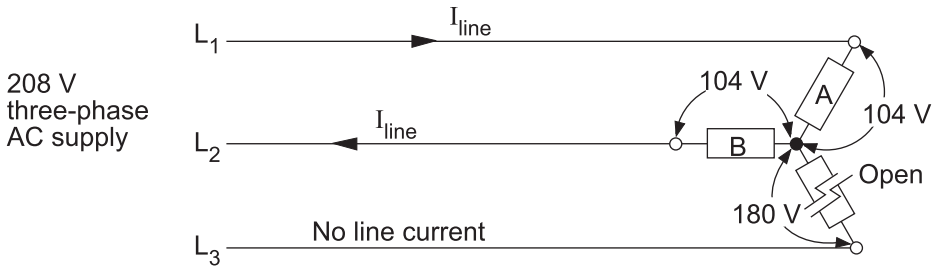


Figure 4—An open in the load phase

The voltage across the open in Figure 4 is 180 V. Study the voltage vector diagram in Figure 5 to see why this is so.

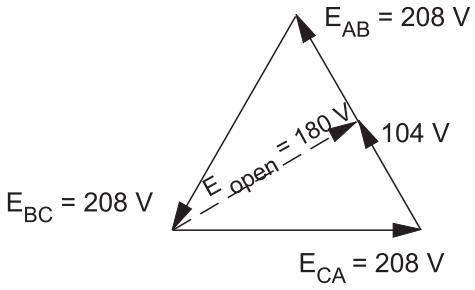


Figure 5—Voltage vector diagram of the circuit in Figure 4

Neutral connected to the load

If the neutral conductor is connected to the load, as illustrated in Figure 6, the voltage across each of the two good phases is again out of phase by 120°, and ⅔ of the previous three-phase power is developed. Compare the circuits of Figures 3 and 6.

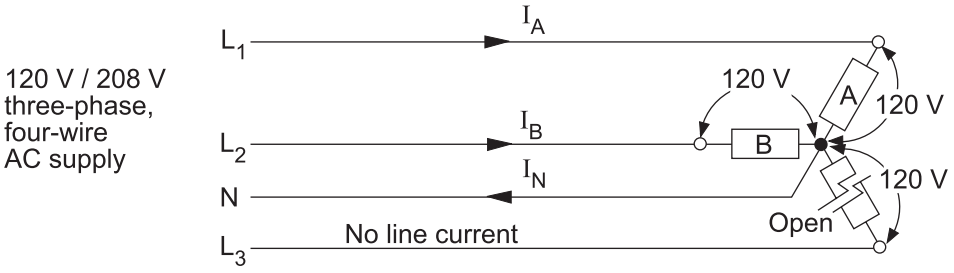


Figure 6—An open load phase with the neutral connected

Observe that, for a three-phase, wye-connected load, an open in the line or an open in the load phase will have the same effect.

Recognizing the changes that occur under an open-fault condition will greatly assist in troubleshooting three-phase circuits.

Delta circuits

The effects of opens on three-phase, delta circuits are similar to wye circuits and are based on where the open occurs.

An open in the line

As described earlier, if an open occurs on one of the line conductors supplying the three-phase load, the condition referred to as *single-phasing* results. This occurs if one of the line fuses blows open (see Figure 7) in a delta circuit, as well.

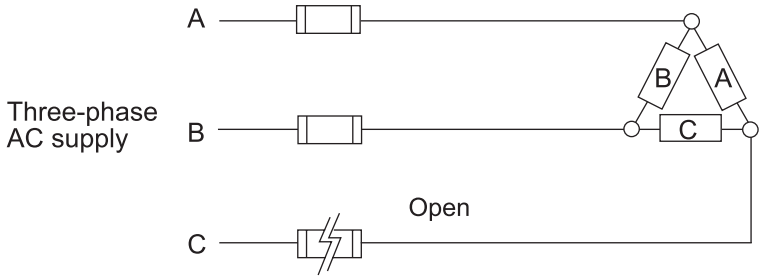


Figure 7—Single-phasing resulting from an open line

Under this condition, effectively, the impedances of two phases are connected in series across one of the three-phase line combinations. In a delta connection, the remaining phase impedance is in parallel with this series combination. Study Figure 8, which is redrawn to show a simplified version of Figure 7.

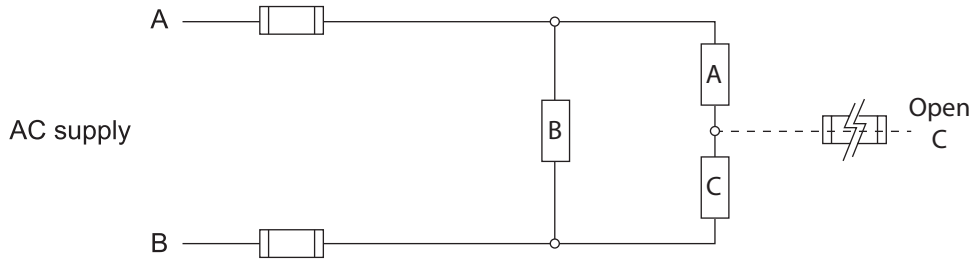


Figure 8—Simplified circuit

An open in one of the lines to a delta-connected load results in a parallel, single-phase circuit with two branches.

Example 2: A three-phase resistance furnace consists of three 10 Ω resistances connected in delta supplied from a 208 V, three-wire supply, as shown in Figure 9. If one of the line fuses feeding this load blows open, what is the:

a. current in each of the line conductors?

b. total power developed?

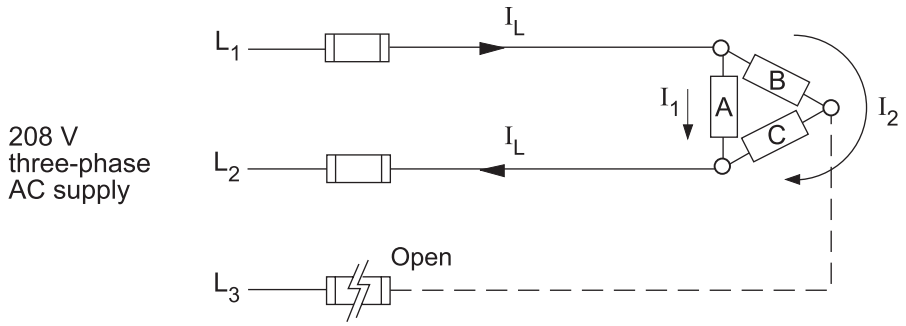


Figure 9—Open line to delta load

Solutions:

a. As illustrated in Figure 9, if line 3 opens, a parallel circuit consisting of two branches results. The total current (I_L) is the sum of the currents through circuit branches 1 and 2.

Calculate the current (I_1) through branch 1 using Ohm's law:

$$I_1 = \frac{E}{R_1} = \frac{208\ V}{10\ \Omega} = 20.8\ A$$

Since branch 2 is a series circuit, its net resistance (R_2) is the sum of the two-phase resistance values:

$$R_2 = 10\ \Omega + 10\ \Omega = 20\ \Omega$$

Calculate the current (I_2) through branch 2 as:

$$I_2 = \frac{E}{R_2} = \frac{208\ V}{20\ \Omega} = 10.4\ A$$

The total line current is the sum of the two branch currents:

$$I_L = I_1 + I_2$$

$$= 20.8 + 10.4 = 31.2\ A$$

Since this is a single-phase circuit, the current flowing in line 1 is the same as the current in line 2. There is no current in line 3 since it is open.

Notice that the voltage drop:

- across phase A resistor is 208 V
- across each of the phase B and phase C resistors is 104 V since they are in series across 208 V

b. There are several different ways to solve for the circuit power. One way is to determine the power developed by each branch and simply add them together for the total power.

For branch 1:

$$P_1 = E \times I_1 \times PF$$
$$= 208\ V \times 20.8\ A \times 1$$
$$= 4330\ W$$

For branch 2:

$$P_2 = E \times I_2 \times PF$$
$$= 208\ V \times 10.4\ A \times 1$$
$$= 2160\ W$$

The total power is

$$P_T = P_1 + P_2$$
$$= 4326.4 + 2163.2\ W$$
$$= 6490\ W$$

This example finds the values under the open-line fault condition known as single phasing. Compare the values in this situation with normal three-phase operating condition:

The normal current in each phase of the delta connection is:

$$I_P = \frac{E_P}{R_P} = \frac{208\ V}{10\ \Omega} = 20.8\ A$$

Since the line current is greater than the phase current, the current in each line is:

$$I_L = \sqrt{3} \times I_P$$
$$= \sqrt{3} \times 20.8 = 36\ A$$

Then, the total three-phase power developed is:

$$P_T = \sqrt{3} \times E_L \times I_L \times PF$$
$$= \sqrt{3} \times 208\ V \times 36\ A \times 1$$
$$= 12980\ W$$

NOTE: The power developed by this three-phase load under a single-phase fault condition drops to approximately 50% or one-half of the power produced in its normal operating condition.

An open in the load

If an open occurs within one of the load phases, this is called an *open-delta load*. The effects are different from those described for single-phasing of a delta load.

In an open-delta load, as shown in Figure 10, two of the load phases continue operating under normal conditions. Since the remaining load phase is open, there is no current flow through this phase—although there is a potential difference across it. No power is produced by this phase.

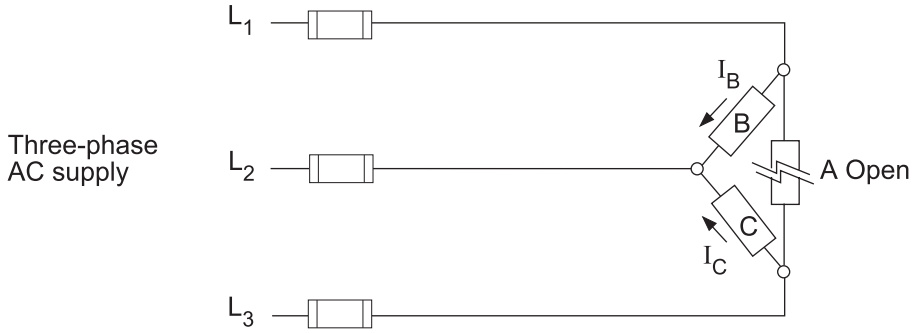


Figure 10—An open-delta load

Since one load phase is lost and two load phases operate as normal, the total power developed by an open-delta load is two-thirds or 66.7% of a normal, three-phase condition.

Example 3: A three-phase resistance furnace consists of three 10 Ω resistances connected in delta supplied from a 208 V, three-wire supply. If one of the load phases develops an open, what is the:

a. current in each of the line conductors?

b. total power produced?

A diagram is extremely helpful in all problem solving. Figure 11 has been simplified by leaving the resistance of phase A out of the previous diagram completely, since it is considered open and conducts no current ($I_A = 0$).

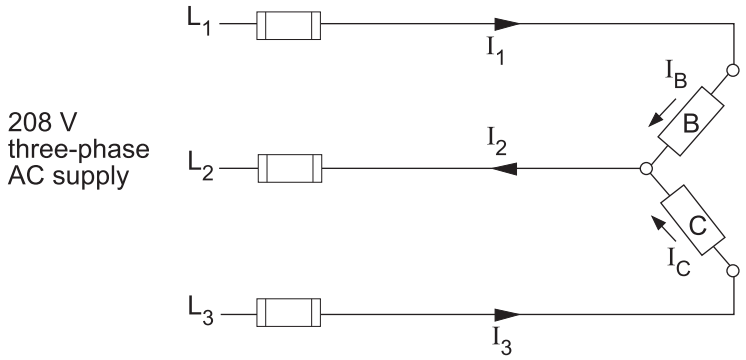


Figure 11—Diagram for Example 3

Solutions:

a. To determine the line currents in an open-delta load, first calculate each of the phase currents:

$$I_B = I_C = \frac{E}{R} = \frac{208\ V}{10\ \Omega} = 20.8\ A$$

Notice that the current in line 1 must be the same current that flows in phase B of the load:

$$I_1 = I_B = 20.8\ A$$

Also, the current in line 3 must be the same current that flows in phase C of the load:

$$I_3 = I_C = 20.8\ A$$

However, the current in line 2 is the vector sum of both the currents of phase B and phase C. Since these phase currents are equal in magnitude, this resultant line current is simply determined by using the fundamental current equation for a delta bank:

$$I_L = \sqrt{3} \times I_P$$

$$= \sqrt{3} \times 20.8 = 36\ A$$

Notice that with an open-delta load situation, two of the line currents drop to a value equal to the phase currents, and the third line current remains the same as before.

b. As before, the easiest way to determine the total power is to add the power produced in each phase. Since these two phases are equal, the power is calculated as:

$$P_T = 2 \times P_P$$

$$= 2 \times E_P \times I_P \times PF$$

$$= 2 \times 208\ V \times 20.8\ A \times 1$$

$$= 8653\ W$$

This value of power represents two-thirds or 66.7% of the power a balanced delta-connected load would produce.

The line voltage and phase impedance values used for Examples 2 and 3 were purposely the same. Review the values calculated and compare them to the values of a normal three-phase-bank operation. You should be able to draw some simple conclusions that may assist you in troubleshooting simple three-phase circuits.

 Now do Self-Test 6 and check your answers.

Self-Test 6

1. Three identical resistors are wye-connected to a three-phase, 220 V line, as shown in Figure 1. If 10 A flow in each line conductor, determine the:

   a. resistance of each load phase

   b. total power developed

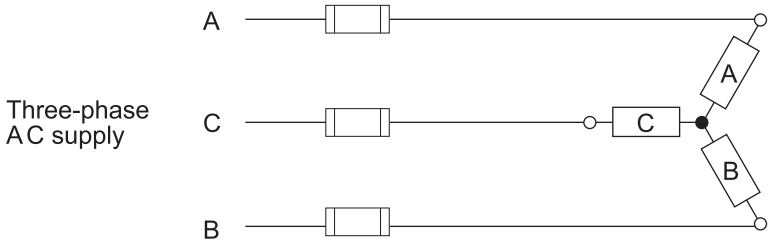


Figure 1—Wye-connected load

2. In Figure 1, if the load marked "A" burns open, determine the:

   a. current in each line

   b. total power developed

3. With all three loads functioning properly, if the fuse in line A blows open, determine the:

   a. current in each line

   b. total power developed

4. When a three-wire, wye-connected load single phases, what percentage of power is developed compared to its normal, three-phase value?

5. Three identical resistors are delta-connected to a three-phase, 220 V supply, as shown in Figure 2. If 10 A flows in each line conductor, determine the:

   a. resistance of each load phase

   b. total power developed

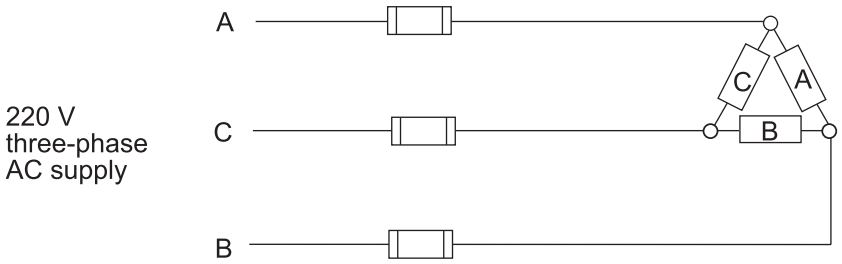


Figure 2—Delta-connected load

6. If the load marked "A" in Figure 2 burns open, determine the:

   a. current in each line

   b. total power developed

7. If all three loads in Figure 2 are functioning properly and the fuse in line A blows open, determine the:

   a. current in each line

   _____

   b. total power developed

   _____

8. What percentage of the normal three-phase power value is developed in a delta-connected load under each of the following open-fault conditions?

   a. a blown line fuse or single-phasing

   _____

   b. an open delta-load condition

   _____

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 7:

Calculate power and power factor in balanced three-phase systems

Power factor in balanced three-phase systems

In single-phase AC circuits, the power factor of the circuit is the ratio of the true power to the apparent power. The same is true in three-phase circuits. The ratio of true power to apparent power is equal to the cosine of the phase angle between the phase voltage and the phase current in each phase. Normally, this angle is symbolized by the Greek letter theta, θ.

$$PF = \cos \theta$$

Power in a balanced three-phase system

In a three-phase system, total power is equal to the sum of the power dissipated in each of the three loads.

In a balanced three-phase system, the power in each phase is the same. Therefore, total power may be obtained by calculating the power in one phase and multiplying it by three.

The power per phase in a three-phase system is:

$$P_P = E_P \times I_P \times PF$$

Therefore, total power in any balanced three-phase system is:

$$P_T = 3 \times E_P \times I_P \times PF$$

Note that this equation can only be used for balanced three-phase systems where the load in each phase is equal in magnitude and has the same power factor. This formula can be expressed in terms of line voltage and line current rather than in terms of phase voltage and phase current.

The Y-connected system

In a Y-connected system, the line current is equal to the phase current and the line voltage is equal to $\sqrt{3}$ times the phase voltage.

Therefore,

$$I_P = I_L \quad \text{and} \quad E_P = \frac{E_L}{\sqrt{3}}$$

Substituting these values into the previous total power equation gives the following equation:

$$P_T = 3 \times \frac{E_L}{\sqrt{3}} \times I_L \times PF$$

$$= \sqrt{3} \times E_L \times I_L \times PF$$

The Δ-connected system

In a Δ-connected system, the line voltage is equal to the phase voltage and the line current is equal to $\sqrt{3}$ times the phase current.

Therefore:

$$E_P = E_L \text{ and } I_P = \frac{I_L}{\sqrt{3}}$$

Substituting these values into the previous total power equation gives the following equation:

$$P_T = 3 \times E_L \times \frac{I_L}{\sqrt{3}} \times PF$$

$$= \sqrt{3} \times E_L \times I_L \times PF$$

Notice that the equation for calculating total power in a balanced three-phase system is the same for both Y and Δ systems.

Solved problems

Example 1: What power is dissipated by a balanced, purely resistive, Y-connected system if the line current is 20 A and the line voltage is 208 V?

$$P_T = \sqrt{3} \times E_L \times I_L \times \cos\theta$$

$$= \sqrt{3} \times 208 \text{ V} \times 20 \text{ A} \times 1$$

$$= 7200 \text{ W}$$

Example 2: A current of 30 A flows through the line conductors of a 240 V, balanced, Δ-connected system. The phase angle between the phase voltage and the phase current is 40°. What is the true and apparent power in this system?

$$VA_T = \sqrt{3} \times E_L \times I_L$$

$$= \sqrt{3} \times 240 \text{ V} \times 30 \text{ A}$$

$$= 12471 \text{ VA}$$

$$P_T = VA \times \cos\theta$$

$$= 12\,471 \times \cos 40°$$

$$= 9553 \text{ W}$$

Example 3: A balanced, purely resistive, Y-connected system has a phase voltage of 277 V and a line current of 15 A. What is the power in this system?

$$P_T = 3 \times E_P \times I_P \times \cos \theta$$

$$I_P = I_L$$

$$P_T = 3 \times 277 \text{ V} \times 15 \text{ A} \times 1$$

$$= 12\,465 \text{ W}$$

Three-phase power measurement using wattmeters

Although commercial or industrial three-phase power may be measured by using three-phase wattmeters, you will most likely be using single-phase wattmeters to measure power consumption in three-phase circuits.

Three-wattmeter method

When you must monitor the power taken by each of three phases, use three single-phase wattmeters. Connect the current coils of the wattmeters in series with the three line conductors. Connect the potential coils of each wattmeter from their respective lines to the neutral conductor. When there is no neutral, *common together* (electrically join) the ends of the potential coils to form a wye as shown in Figure 1.

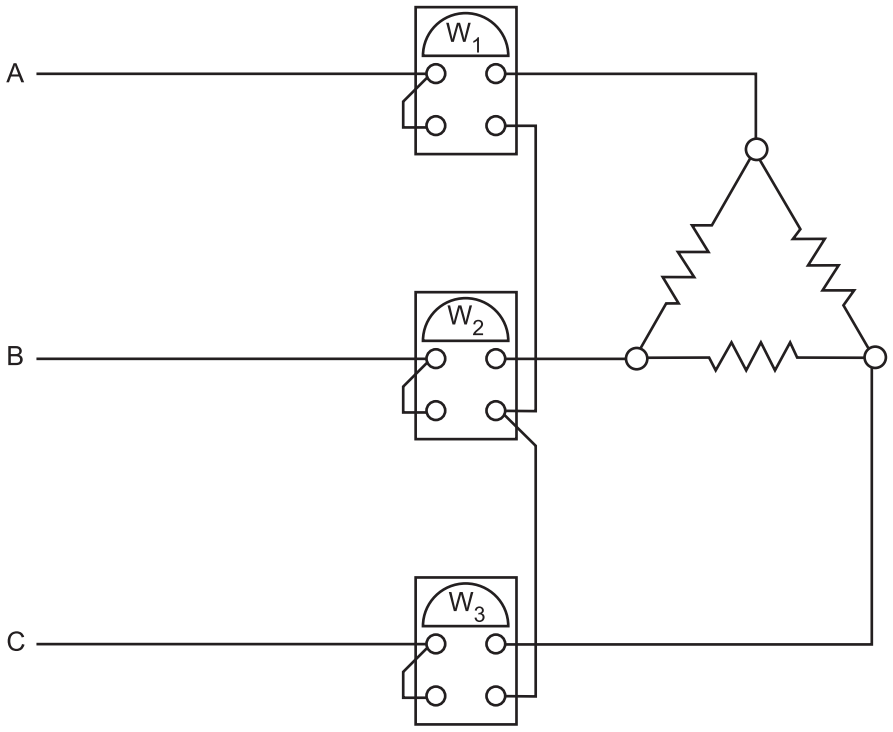


Figure 1—Three-wattmeter method of three-phase power measurement

For accurate power measurement, the potential coils of each wattmeter must have the same impedance. Also, each wattmeter must be connected with proper polarity. Total three-phase power is the sum of the three individual wattmeter readings:

$$P_T = W_1 + W_2 + W_3$$

Although this method of power measurement is typically used for three-phase, four-wire, wye systems, it can also be used for three-phase, three-wire, delta systems as shown.

Two-wattmeter method

Another common method of three-phase power measurement uses only two wattmeters, and may be applied to wye- or delta-connected loads. In this case, connect the current coils of the wattmeters in series with *two* of the line conductors. Connect each wattmeter's potential coil between the respective current-coil line and the remaining line conductor as shown in Figure 2.

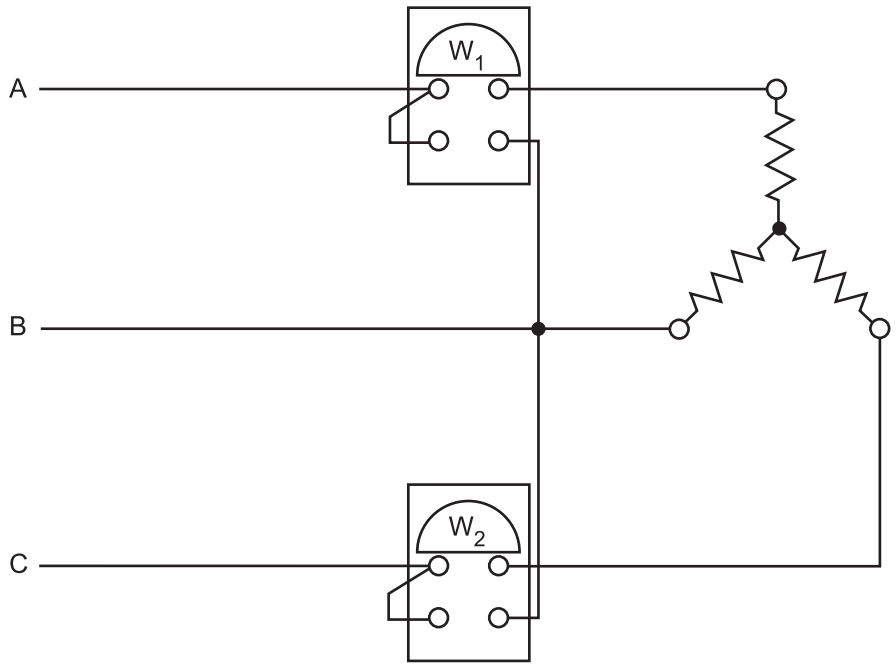


Figure 2—Two-wattmeter method of power measurement

Total three-phase power is the sum of the two individual wattmeter readings:

$$P_T = W_1 + W_2$$

For a balanced three-phase load, the load power factor affects the wattmeter readings as follows:

PF value	Effect on wattmeter readings
100%	Each wattmeter reading is one-half of the total three-phase power. $W_1 = W_2$
50% to 100%	One wattmeter reads higher than the other. $W_1 > W_2$
50%	One wattmeter reads zero, and the other reads the total power. $W_1 = 0$ and $W_2 = P_T$
up to 50%	One wattmeter tends to read downscale. To obtain an upscale reading, reverse connections to the potential coil of this wattmeter. Subtract the reading of this reversed-polarity wattmeter from the other wattmeter's reading. $P_T = W_2 - W_1$

The two-wattmeter method of power measurement also allows you to determine the power factor of a *balanced*, three-phase load by using the following formula:

$$\text{Tan } \theta = 1.732 \frac{(W1 - W2)}{(W1 + W2)}$$

$$\theta = \text{Tan}^{-1} 1.732 \frac{(W1 - W2)}{(W1 + W2)} \text{ where cos } \theta = \text{PF of load}$$

Single-wattmeter method

For balanced, three-phase loads in which connections can be made into each phase, you may use a single wattmeter. Determine total power by multiplying the wattmeter reading by three. Note that this method of three-phase power measurement is of limited usefulness.

Now do Self-Test 7 and check your answers.

Self-Test 7

1. What is the mathematical equation for calculating the true power in a balanced, three-phase, Y-connected system?

2. What is the mathematical equation for calculating the true power in a balanced, three-phase, Δ-connected system?

3. What is the mathematical relationship between the line current and the phase current in a balanced, Δ-connected system?

4. What is the mathematical relationship between the line voltage and the phase voltage in a balanced, Y-connected system?

5. Calculate the total power dissipated in a purely resistive, balanced, Y-connected system when the line voltage is 480 V and the line current is 20 A.

6. Calculate the true and apparent power of a balanced, Δ-connected system when the line voltage is 480 V and the line current is 40 A. The phase current in this system lags the phase voltage by 30°.

7. What line current is flowing in a balanced, purely resistive, Δ-connected system if the line voltage is 480 V and the true power in the system is 49 883 W?

8. What is the phase voltage of a balanced, purely resistive, Y-connected system if the line current is 25 A and the total power is 20 785 W?

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 8:

Describe the connection of capacitors for power factor correction in three-phase circuits

As you will recall from single-phase studies, the ideal transmission of electrical energy in AC systems occurs when the current is in phase with the voltage—that is, when the power factor is unity. With unity power factor, the amount of line current required for any fixed amount of true power is minimum. Remember, it is the *true power*, measured in watts, that is the practical measurement of useful work performed in an electrical circuit.

The most common types of low-power factor loads in large commercial and industrial installations are induction motors and ballast-type lighting. As shown in Figure 1, the kVA load of these devices has two components: the true (or active) power, which does the work; and the lagging reactive power, which magnetizes the device and causes an increase in the kVA demand.

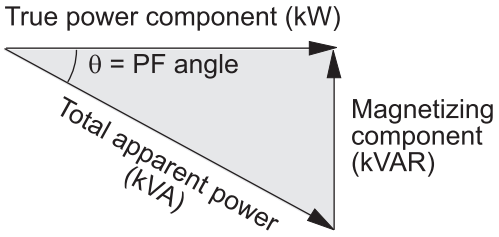


Figure 1—Power factor diagram for inductive loads

Lagging power factor can be corrected by connecting capacitors in parallel with either the inductive loads or the system. Unity PF is achieved by selecting a value of capacitance that will provide a value of leading kVAR equal to the lagging kVAR of the load. In most practical applications, the inductive power factor is corrected to a value of about 90% lagging. Remember that the kW component is virtually unaffected by the addition of capacitors to the circuit, because they have negligible resistance. The capacitor delivers *reactive power* to the load to reduce the amount of kVAR that the source normally supplies. The resulting PF diagram is shown in Figure 2.

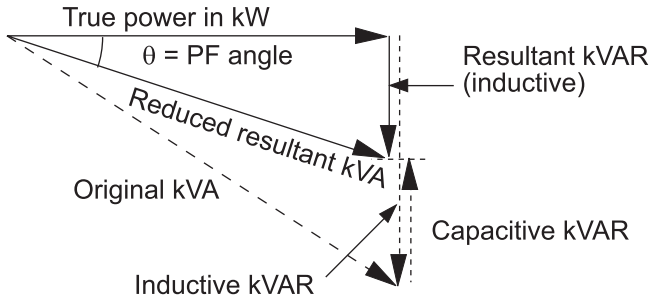


Figure 2—Power factor diagram after addition of capacitors

Methods of correcting power factor

There are three basic methods used for correcting power factor:

Individual Capacitor Units. This is where one capacitor is installed at each inductive load. This is the best method for improving overall power factor in a distribution system, since the capacitor and the load can be switched on and off together. This provides lower line losses, reducing heat damage caused by larger currents. However, this method increases the overall installation costs of both labour and materials.

Large Bank of Capacitor Units. This involves the connection of several capacitors at the central distribution point of the system. This is the most economical method of installating capacitors to reduce the overall system power factor. However, it does require the switching of capacitors based on the load requirements. Switching prevents any over-capacitance from creating overvoltages whenever the net inductive load is decreased.

Combination of Individual Units and Larger Banks. This is where individual capacitors are installed at large motors, and banks of capacitors are installed on one or more main feeders, or switchboards. This involves knowing the net inductive kVAR on the system and determining the desired overall power factor.

Construction of three-phase PF correction capacitors

PF correction on three-phase systems can be achieved by connecting three single-phase capacitors as a three-phase bank, or by using a three-phase capacitor unit. Three-phase capacitors are typically more economical, as well as physically smaller, than three single-phase capacitors of the same equivalent kVAR rating.

Three-phase, PF correction capacitors are constructed internally as either wye-connected banks or delta-connected units, as shown in Figure 3. Wye-connected units are more commonly used for high-voltage applications to reduce the phase-insulation requirements. Delta-connected units are used for larger-current-capacity installations to reduce phase-ampacity ratings. Regardless of their internal configuration, the three terminal leads are connected to the supply-line conductors so that the capacitor is simply in parallel with the circuit load.

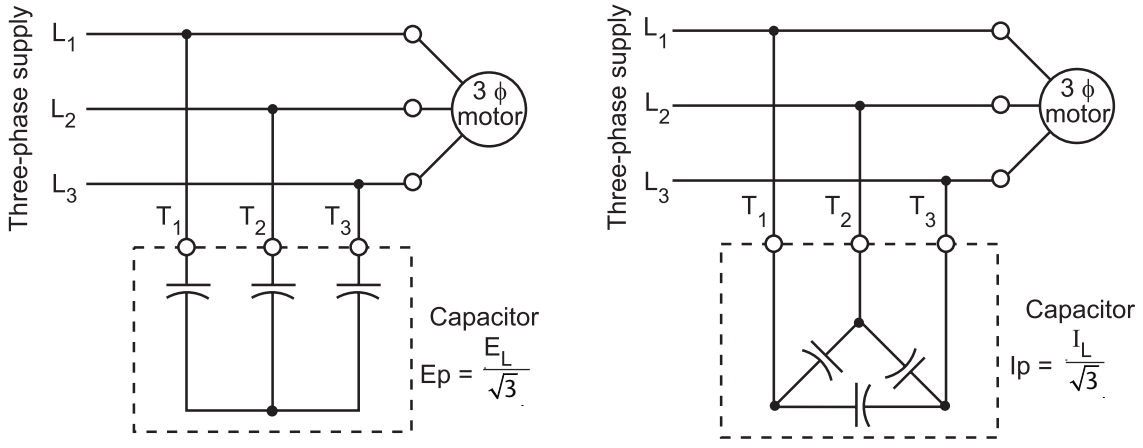


Figure 3—Internal connection of three-phase, PF correction capacitors

Connection of PF correction capacitors

Figure 4 illustrates the various locations that the capacitor unit can be connected in the low voltage distribution system.

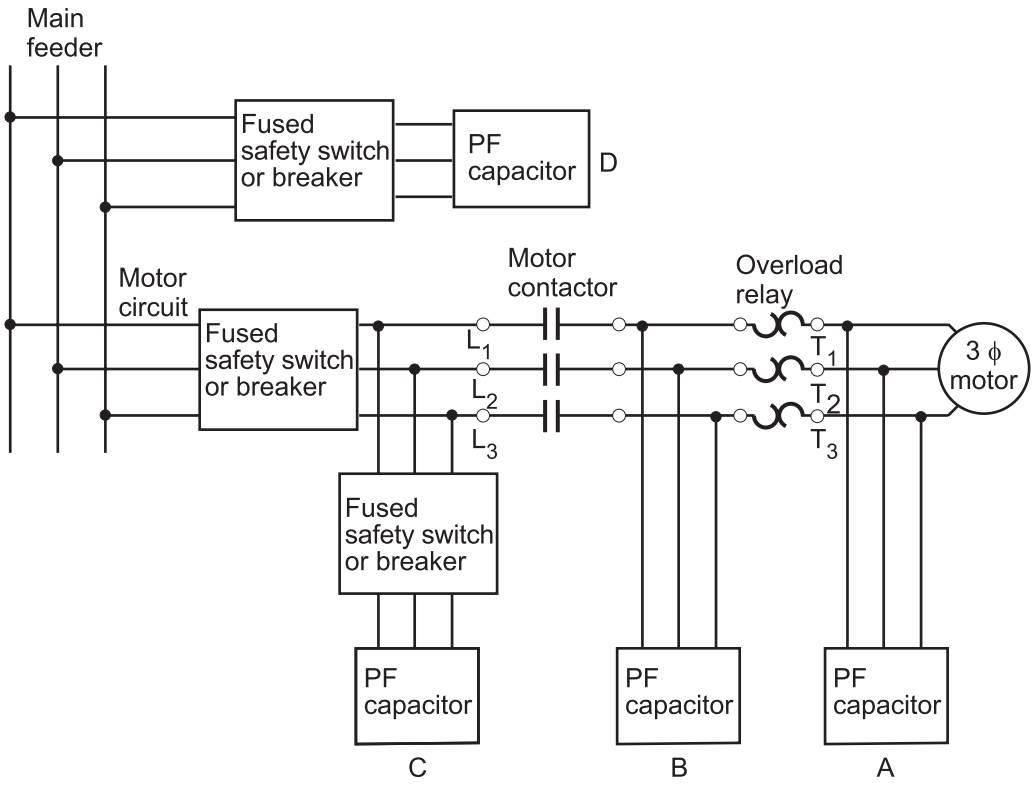


Figure 4—Locations for capacitors in motor circuits

Location A in Figure 4 illustrates the most efficient location for connecting the capacitor. Since the capacitor is connected directly across the inductive motor load, the capacitor is switched automatically by the motor starter so it is only energized whenever the motor is running.

Care must be taken in properly sizing the overload relay heaters to the *reduced line current* passing through the overload relay (as a result of PF correction) when connecting the capacitor at the motor.

Location B in Figure 4 offers similar advantages as Location A, except that there is no need to de-rate the overload relay protection. This location is often preferred by switchgear and motor control centre manufacturers.

Location C in Figure 4 is usually recommended for motor applications involving jogging, plugging and reversing. In this location, the capacitor unit is not switched by the motor starter.

Location D in Figure 4 illustrates the connection point for a larger bank of capacitors for correcting the PF on the feeder. This is usually the most cost-efficient, since it uses a bank of large capacitors rather than many smaller units at the individual loads. However, the possibility of over-correcting the PF does exist unless the bank is provided with selective control.

Safety precautions with capacitors

From past studies, you know that certain capacitors can store a charge for a considerable period of time. Most manufacturers of PF correction capacitors provide discharge (bleeder) resistors across the internal terminals of the capacitor, so as to drain the capacitor of harmful charges when the capacitor is disconnected from the power source.



Low-voltage PF capacitors usually require at least one minute to discharge. After this, the capacitor should be tested for voltage, and the terminals ALWAYS short-circuited to ground before handling.

The CEC requires that the disconnect means for capacitor feeders or branch circuits have a warning notice such as:

WARNING - CAPACITOR CIRCUIT

Wait 5 minutes after opening, then short capacitors before handling.

- Capacitors should be located in areas where there is adequate ventilation, and the ambient temperature does not exceed 40°C in order to prevent overheating.
- Capacitors should be operated at their nameplate frequency and voltage ratings. Never operate a capacitor beyond its voltage rating, since overheating can occur.
- Never over-size capacitors or exceed unity (100%) power factor, since this can create problems with the motor.
- Refer to the manufacturer's data for the proper application and installation of capacitors.



Now do Self-Test 8 and check your answers.

Self-Test 8

1. There are several methods of connecting capacitors for PF correction: individual units can be connected at the load; a large bank can be connected at the main service; or a combination of individual units and large banks can be used.

   a. Which method is usually preferred, and why?

   _____

   b. Which method is usually considered the most economical?

   _____

2. Three-phase PF correction capacitors can be either wye- or delta-connected internally. Which connection is preferred for:

   a. higher voltage installations

   _____

   b. higher current applications

   _____

3. When connecting a PF correction capacitor directly at the motor, what important consideration must be observed for the motor starter?

   _____

   _____

   _____

   _____

4. What safety precautions must be observed before working on capacitors or their circuits?

   _____

   _____

   _____

   _____

5. For a disconnect switch supplying a capacitor only, what important marking is required?

6. To what value of PF are most inductive loads usually corrected?

Go to the Answer Key at the end of the Learning Guide to check your answers.

Learning Task 9:

Calculate the ratings of capacitors for three-phase PF correction

Earlier, you learned how to apply the power triangle method to determine various power components (watts, VARs and VA, as well as power factor) in an AC circuit. The process used to determine the ratings for PF correction capacitors in three-phase installations is similar to that for single-phase circuits. The difference is that in three-phase circuits, you must use formulas that deal with line and phase value relationships.

Example 1: A motor load of 60 kW with an average power factor of 70% lag is connected to a 240 V, three-phase supply. Determine the:

a. three-phase line current

b. kVAR rating of a three-phase capacitor to correct the overall PF to 100%

c. line current after correction to 100% PF

d. kVAR rating of a three-phase capacitor to correct the overall power factor to 90% lag

e. line current after correction to 90% PF

Solutions:

a. To calculate the line current for this three-phase load, you can use, $P_T = \sqrt{3} \times E_L \times I_L \times PF$, first transposing it:

$$I_L = \frac{P_T}{\sqrt{3} \times E_L \times PF}$$

$$= \frac{60\,000\ W}{\sqrt{3} \times 240\ V \times 0.7} = 206\ A$$

b. Since you are given the kW and the PF, you can determine the total kVA and kVAR by solving the power triangle for this load as follows (see Figure 1):

$$PF = \cos\theta = 0.7$$

$$\theta = 45.6°$$

$$kVA = \frac{kW}{\cos\theta} = \frac{60}{0.7} = 85.7\ \ kVA$$

$$kVAR = \sqrt{kVA^2 - kW^2}$$

$$= \sqrt{85.7^2 - 60^2} = 61.2\ kVAR, lagging$$

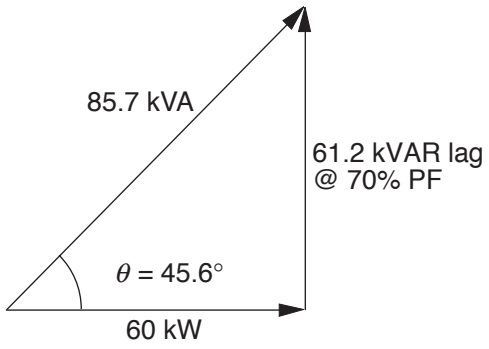


Figure 1—Power triangle for Example 1

To achieve 100% PF, the capacitor must provide as many leading kVAR as the load has lagging kVAR. In this case, the capacitor has a calculated rating of 61.2 kVAR leading.

c. After correction to 100% PF, the new kVA is equal to the kW. The new line current at unity PF is:

$$I_L = \frac{VA}{\sqrt{3} \times E_L} = \frac{60\,000\ VA}{\sqrt{3} \times 240\ V} = 144\ A$$

d. When a capacitor is used to correct the PF to only 90% lag, there is still some lagging kVAR present. Since the kW is unaffected by the addition of the capacitor, you can determine the remaining inductive kVAR in this new power triangle.

$$PF = \cos\theta = 0.9$$

$$\theta = 25.8°$$

$$kVA = \frac{kW}{\cos\theta} = \frac{60}{0.9} = 66.7\ kVA$$

$$kVAR = \sqrt{kVA^2 - kW^2}$$

$$= \sqrt{66.7^2 - 60^2} = 29.1\ kVAR, lagging$$

To calculate the rating of the capacitor needed for correction to 90% PF, subtract the remaining lagging kVAR from the original value:

Capacitor kVAR = Original kVAR – Remaining kVAR

= 61.2 kVAR lag – 29.1 kVAR lag

= 32.1 kVAR lead

e. After correcting to 90% PF, the kW component remains the same. The new line current is:

$$I_L = \frac{W}{E_L \times \sqrt{3} \times PF}$$

$$= \frac{60\,000}{240 \times \sqrt{3} \times 0.9} = 160.4 \text{ A}$$

Ratings for power factor correction capacitors

Low-voltage capacitors for power factor correction are available for single-phase and for three-phase, 60 Hz applications. Typically, manufacturers provide standard values from 2 kVAR to 100 kVAR at voltages of 208, 240, 480 and 600 V.

Nameplate information includes the manufacturer's name, the kVAR rating, the single-phase or three-phase rms voltage rating, and the frequency rating. Usually the current rating is not listed, and you will have to calculate this in order to determine ratings of conductors, switches and overcurrent protection for capacitors.

Example 2: Determine the rated capacitor current for a 10 kVAR, 480 V, 60 Hz, three-phase capacitor.

For practical calculations, three-phase capacitors are treated as purely reactive loads, and as such the kVA = kVARs and kW = 0.

Solution:
To calculate the line current to this capacitor, transpose the formula:

$$VAR = \sqrt{3} \times E_C \times I_C$$

$$I_C = \frac{VAR}{\sqrt{3} \times E_C} = \frac{10\,000 \text{ VAR}}{\sqrt{3} \times 480 \text{ V}} = 12 \text{ A}$$

This rated current of 12 A is used to determine the various installation requirements for the capacitor, including the following:

a. conductor size

b. overcurrent protection

c. disconnect switch rating

You will apply this later.

Now do Self-Test 9 and check your answers.

Self-Test 9

1. An industrial plant has a total load of 750 kVA at an average power factor of 70% lagging. Determine the size of capacitor necessary to improve the power factor to:

   a. 100%

   b. 90% lagging

2. If the plant load of question 1 is supplied by a 480 V, three-phase supply, determine the line current:

   a. before correction

   b. after correction to 100% PF

   c. after correction to 90% PF

3. Determine the capacitor current for question 1 if the power factor is corrected to:

   a. 100%

   b. 90% lagging

4. When a 300 kVAR capacitor is connected across a 650 kW, three-phase load, the overall power factor is corrected to 90%. What was the original:

   a. PF before the capacitor was installed

   b. kVA load

Go to the Answer Key at the end of the Learning Guide to check your answers.

Answer Key

Self-Test 1

1. Polyphase systems are systems that have two or more phases.

2. • Smaller machines for the same kVA capacity
   • Less copper required to supply the same energy
   • Constant power as opposed to pulsating power in single-phase systems
   • Simpler motor design

3. 120°

4. Y (wye or star) or Δ (delta)

5. Phase sequence is the order in which the three-phase voltages reach their positive peaks.

6. Interchange any two line leads.

7. Phase A = red
   Phase B = black
   Phase C = blue
   Neutral = white or natural grey

8. effective (rms)

9. counter-clockwise

10. Yes (two sine waves having the same frequency add to form a third sine wave with the same frequency)

11. open

12. positive

13. negative

14. enters

15. 180

Self-Test 2

1. 520 volts

2. 231 volts

3. 30°

4. $I_{line} = I_{phase}$

5. $E_{line} = \sqrt{3} \times E_{phase}$

6. Any four of the following:
   - reduction in the magnitude of transient overvoltages
   - improved lightning protection
   - easier location and correction of ground faults
   - more effective system and equipment fault protection
   - extended life of electrical insulation
   - increased personnel and equipment safety

7. 480 V $\angle$ 90°

Self-Test 3

1. 240 volts

2. 346 amperes

3. 30°

4. $I_{line} = \sqrt{3} \times I_{phase}$

5. $E_{line} = E_{phase}$

6. Ungrounded

7. Phase A

8. No (the CEC requires that the high leg be separated from the grounded conductor)

Self-Test 4

1. a. 600 V
   b. 24 kW
   c. 30 kVA
   d. 18 kVAR

2. a. 480 V
   b. 30 A
   c. 52 A
   d. 43.2 kVA

3. a. 69.3 A
   b. 57.6 kW

4. a. 30 A
   b. 52 A
   c. 20 Ω

5. The three phases in a balanced load have the same values of impedance and power factor, and the same value of voltage impressed across them. Unbalanced loading can be caused by a fault condition or by the addition of a single-phase load.

6. a. 10 A
   b. 10 A
   c. 3.4 kW

7. a. 347 V
   b. 28.7 kW

8. a. 5.7 A
   b. 10 A
   c. 23 A

9. a. 4 Ω
   b. 1.3 Ω

10. a. 69 kW
    b. 23 kW

Self-Test 5

1. a. 50 A
   b. 66 A
   c. 13 A

2. $I_A = 50$ A
   $I_B = 33.3$ A
   $I_C = 20.8$ A
   $I_N = 25.4$ A

3. 40 A
   30 A
   24 A
   $I_N = 14$ A

4. 10 A

5. a. 8.66 A
   b. 13.2 A

Self-Test 6

1. a. 12.7 Ω
   b. 3.8 kW

2. a. $I_B = I_C = 8.6$ A
      $I_A = 0$
   b. 1.9 kW

3. a. $I_B = I_C = 8.6$ A
      $I_A = 0$
   b. 1.9 kW

4. 50%

5. a. 38 Ω
   b. 3.8 kW

6. a. $I_A = I_B = 5.8$ A
   $I_C = 10$ A
   b. 2.54 kW

7. a. $I_B = I_C = 8.7$ A
   $I_A = 0$
   b. 1.9 kW

8. a. 50%
   b. 66.7%

Self-Test 7

1. $\sqrt{3} \times E_L \times I_L \times PF$

2. $\sqrt{3} \times E_L \times I_L \times PF$

3. $I_L = \sqrt{3} \times I_P$

4. $E_L = \sqrt{3} \times E_P$

5. 16 628 W

6. $P_T = 28\,800$ W
   $VA_T = 33\,255$ VA

7. 60 A

8. 277 V

Self-Test 8

1. a. *Individual units at the load* so that the correcting capacitor is switched with the inductive load to prevent over-correcting the system.

   b. *A large bank at the main service* to save on labour and materials.

2. a. wye-connected units

   b. delta-connected units

3. The overload relay protection (heaters) must be de-rated to the reduced line current value as a result of PF correction.

4. Wait at least one minute (or five minutes for capacitor-only circuits); then test for any voltage present; then short-circuit the capacitor terminals to ground.

5. A "warning notice" must be affixed to the disconnecting means indicating that the circuit has capacitors, and that a waiting period of five minutes be observed, after which the capacitors shall be discharged before handling.

6. Between unity 90% lagging

Self-Test 9

1. a. 535.6 kVAR

   b. 535.6 – 254.3 = 281.3 kVAR

2. a. 902 A

   b. 631 A

   c. 702 A

3. a. 644 A

   b. 338 A

4. a. 73%

   b. 895 kVA



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