

BC CARPENTER APPRENTICESHIP PROGRAM **LEVEL 1**

Line B: Documentation and Organizational Skills

Competency B-4: Perform Trade Math

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Version 2, New, January 2017

ISBN 978-0-7726-7030-4

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Acknowledgments

The Industry Training Authority of British Columbia would like to acknowledge the Carpentry Articulation Committee and Open School BC, a division of the BC Ministry of Education, as well as the following individuals and organizations for their contributions in updating the BC Carpenter Apprenticeship Learning Guides:

Carpentry Articulation Curriculum Committee members

Dennis Carlson, Tom Haag, Erik Hardin, Alf Leimert, Geoff Murray, Don Naidesh, Stephen Pelley, Al van Akker

Writers: Gary Backlund, Geoff Murray

Reviewers: Trevor Feddersen, Roy Mironuck, Geoff Murray, Don Naidesh, Stephen Pelley

Open School BC

Christina Teskey, project management

Dennis Evans, photography, illustration

Farrah Patterson, print layout, illustration

Shannon Sangster, copyright management, art coordination

Beverly Carstensen, print layout, illustration

Max Licht, illustration

Greg Aleknevicus, editing

Robin Miller, editing

Ordering

Crown Publications, Queen's Printer

PO Box 9452 Stn Prov Govt

Victoria, BC V8W 9V7

Phone: 1 800 663-6105

Fax: 250 387-1120

Email: crownpub@gov.bc.ca

Web: www.crownpub.bc.ca

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Symbol Legend



Electric shock: This icon is a reminder for potential electric shock.



Explosive: This icon is a reminder for a possibly explosive situation.



Flammable: This icon is a reminder for a potentially flammable situation.



Important: This icon highlights important information.



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We want your feedback! Please go to the BC Trades Modules website (www.bctradesmodules.gov.bc.ca) to enter comments about specific sections that require correction or modification. All submissions will be reviewed and considered for inclusion in the next revision.

Program Outline

Line A – Safe Work Practices

A-1 Apply Shop and Site Safety Practices

A-2 Apply Personal Safety Practices

Line B – Documentation and Organizational Skills

B-1 Use Construction Drawings and Specifications

B-2 Interpret Building Codes and Bylaws

B-3 Plan and Organize Work

B-4 Perform Trade Math

Line C – Tools and Equipment

C-1 Use Hand Tools

C-2 Use Portable Power Tools

C-3 Use Stationary Power Tools

Line D – Survey Instruments and Equipment

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Line E – Access, Rigging, and Hoisting Equipment

E-1 Use Ladders, Scaffolds and Access Equipment

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G-1 Use Concrete Types, Materials, Additives and Treatments

G-2 Select Concrete Forming Systems

G-3 Build Footing and Vertical Formwork

G-4 Build Slab-On-Grade Forms and Suspended Slab Forms

G-5 Install Reinforcement and Embedded Items

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H-1 Describe Wood Frame Construction

H-2 Select Framing Materials

H-3 Build Floor Systems

H-5 Build Stair Systems

H-10 Build Decks and Exterior Structures

Line J – Building Science

J-1 Control the Forces Acting on a Building

Competency B-4: Perform Trade Math

Fundamental to many aspects of carpentry is the use of mathematics. Preparing drawings requires the use of scale, ratio and proportion; estimating requires the careful summation of many parts, waste and cost factors; site and building layout requires the use of triangles and on centre spacing; roofs and stairs require similar triangle processes; conversion between imperial and metric systems is required when using the building code.

These learning tasks are an introduction to, and a resource for, the many applications of mathematics you will use throughout your apprenticeship.

Learning Objectives

When you have completed the Learning Tasks in this Competency, you will be able to:

- describe and use arithmetic, geometry, simple algebra, simple trigonometry

Competencies

Written: "Use safe work practices"

You will be tested on your ability to apply trade math concepts in the various content areas in the program (i.e. concrete formwork, framing, etc.).

LEARNING TASK 1

Describe Trade Math Concepts

The carpentry trade requires workers to be able to use mathematical concepts. There are many branches of mathematics, but only a small number of these apply to the carpentry trade and most are fairly simple to master. Carpenters focus their attention primarily on arithmetic, geometry, simple algebra, and trigonometry.

Besides being able to add, subtract, multiply, and divide, you also need to know how to convert imperial and metric measurements. While most of this can be done using a calculator, it may not always be practical to use one on the job site. When measuring, remember that any measurement is made up of three things: the numbers, their units, and the degree of accuracy (significant figures). For example, if you're told a property is 63 long in frontage, it's important to know whether the measurement is in feet or meters.

Mathematical Operations

Solving problems involving numbers, fractions, decimals, and angles requires an understanding of the terms commonly used in mathematical calculations.

Addition

Addition is the process of combining a number of objects together into a larger collection. For example, adding 5 joist hangers to a box of 100 joist hangers results in a box containing 105 joist hangers.

The symbol for addition is: +

$$5 \text{ hangers} + 100 \text{ hangers} = 105 \text{ hangers}$$

The terms "sum," "summation," and "aggregate" have the same meaning as "addition."

Subtraction

Subtraction is the process of removing a number of objects from a collection. This decreases the number of objects in the collection. For example, if you need 15 joist hangers and take them from a box containing 105 joist hangers, there will be 90 joist hangers left in the box.

The symbol for subtraction is: –

$$105 \text{ hangers} - 15 \text{ hangers} = 90 \text{ hangers}$$

Subtraction can also be used to figure out the difference between two amounts. For example, taking the highway to work is about 20 kilometres, but taking city streets is about 15 kilometres. Driving through the city is 5 kilometres shorter than taking the highway ($20 \text{ km} - 15 \text{ km} = 5 \text{ km}$).

Multiplication

Multiplication is the mathematical process that speeds up the addition of groups with an equal number of objects in them. For example, a foundation plan calls for 5 anchor bolts on each of its 4 sides. To find the total number of anchor bolts needed, you could add them together ($5 + 5 + 5 + 5 = 20$), or you could multiply the number of bolts per side by the number of sides ($5 \text{ bolts} \times 4 = 20 \text{ bolts}$).

There are several different ways to indicate multiplication. The following examples use multiplication to find the area of a rectangle:

The symbol: $\times$

$$6 \text{ m} \times 5 \text{ m} = 30 \text{ m}^2$$

$$A = L \times W \text{ (Area = Length} \times \text{Width)}$$

The symbol: $\cdot$

$$6 \cdot 5 = 30$$

$$A = L \cdot W$$

Parentheses can be used around terms:

$$6(5) = 30$$

$$A = (L)(W)$$

Terms can be placed directly next to each other:

$$6w = 30$$

$$A = LW$$

Note the last method is not used when both terms are numbers. That is, you would never write "6 multiplied by 5" as "65" (for obvious reasons).

Division

Division is the fourth basic mathematical operation. Division is the process of separating a collection of objects into a number of equal groups. For example, a floor area is 480 square feet and a sheet of plywood covers 32 square feet. A minimum of 15 sheets of plywood are needed to cover the floor ($480 \text{ ft}^2 \div 32 \text{ ft}^2 = 15 \text{ sheets}$).

There are three symbols commonly used for division: $\div$, $/$, and $\frac{\quad}{\quad}$

$$10 \div 5 = 2$$

$$10/5 = 2$$

$$\frac{10}{5} = 2$$

Note that division can also be expressed as a ratio when used with a colon symbol (:). For example, the sides of an excavation must not be steeper than a 3:4 (horizontal : vertical) unless shored.

3:4, which is the same as 0.75h:1v

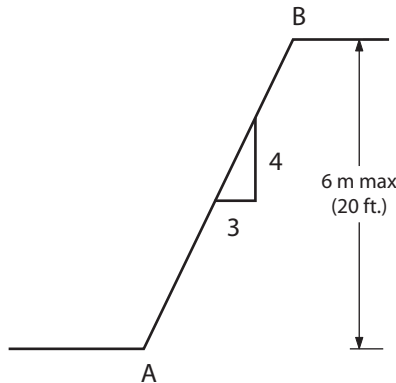


Figure 1 — Sloping at a 3:4 ratio

Exponents

Exponents are used when you multiply the same number multiple times (e.g., 5×5). Exponents are written as a superscript number indicating the number of times the value is multiplied by itself.

$$4^2 = 4 \times 4$$

$$6^3 = 6 \times 6 \times 6$$

Order of Operations

You may be presented with an equation or numerical sentence that includes several operations. It's important to do the calculations in the correct order. The term "BEDMAS" can help you to remember the correct order:

B – brackets

E – exponents

DM – division and multiplication (in the order they occur from left to right)

AS – addition and subtraction (in the order they occur from left to right)

This means that you do anything in brackets (grouping symbols) first, then any exponents, then any division and multiplication (in the order they occur left-to-right), and finally any addition or subtraction (also in the order they occur left-to-right).

If there are several of the same class of operation (e.g., multiplication and division), you do them in order from left to right:

$y = (8 - 2) \cdot 2^2 + 20 \div 5$	Brackets
$y = (6) \cdot 2^2 + 20 \div 5$	Exponents
$y = 6 \cdot 4 + 20 \div 5$	Division and Multiplication (left-to-right)
$y = 24 + 20 \div 5$	Division and Multiplication (left-to-right)
$y = 24 + 4$	Addition and Subtraction
$y = 28$	

Mathematical Equations and Formulas

Using simple equations and formulas to solve problems is a useful skill for all trades. Learning to manipulate equations to determine an unknown quantity takes practice but is essential when solving mathematical problems. Equations are used to convert between metric and imperial measurements, for laying out stairs, for determining rafter lengths, and for calculating materials.

Some key terms follow.

Equation

An equation is a statement of equality between two quantities as indicated by the equal sign (=). All equations must be true. A simple equation will usually have one unknown. (In the examples below, "x" is the unknown.)

$$4x = 5 + 3$$

$$x^2 = 64$$

Formula

A formula is a mathematical law or rule expressed as an equation. For example, the formula for the area of a rectangle is the length times the width. This formula is written as:

$$A = LW$$

Where A = area, L = length, and W = width

A formula has two or more unknown quantities.

Constant

A constant is a numerical value that forms part of an equation or formula. In the equation for finding the perimeter of a square, $P = 4 \cdot L$, the number “4” is a constant because it does not change no matter how large the square.

The formula for the area of a circle is:

$$A = \pi \cdot r^2 \text{ (where } r \text{ is the radius of the circle)}$$

π (pronounced “pie”) is a constant even though it’s not written as a number. It has a fixed numerical value (3.1415...) which does not change no matter how large the circle. π is also used to find the circumference of a circle: $C = \pi \cdot d$ (where d is the diameter of the circle).

Variable

A variable in a formula or equation is a symbol that represents an unknown or non-constant quantity. In the equation $x^2 = 64$, the symbol “ x ” stands for an unknown quantity that can be found by calculation.

In the formula $A = LW$, the symbols “ A ,” “ L ,” and “ W ” are variables that can be measured or calculated (if the value of two variables is known, the third variable can be calculated). Usually letters of the English alphabet are used to represent variables.

Solution

A solution is a value for an unknown quantity that will make an equation true. For example, in the equation $x + 12 = 15$, the solution is $x = 3$.

Solving Word Problems

Complicated mathematical problems can be simplified and solved using a systematic approach. Focussing on one part of a problem will produce the solution more easily and with less chance of error. The following process for solving word problems has four steps:

Step 1 – What are you trying to determine?

There may be several questions within a given problem, so you have to determine which answer will correctly solve the problem.

Step 2 – What numbers are important to solving the problem?

Figuring out which numbers or measurements are going to help find the solution is as important as determining which ones are not.

Step 3 – What mathematical operation is needed to solve the problem?

More than one operation may be needed to get the final result. Numbers may have to be added together (addition) before they are separated into equal groups (division). The possible operations are addition, subtraction, multiplication, and division.

Step 4 – Arrange the given numbers into an equation or number sentence which will result in the final answer.

Your equation should have all the known values on one side of the equal sign, and the unknown value standing alone on the other side.

$$8 + 12 = ?$$

$$4 \times 5 = ?$$

You may wish to do one equation at a time building to the final answer, or combine equations so that one calculation will result in the final answer.

The following example demonstrates how to use the four steps for solving a word problem.

A carpenter requires 112 hours to complete a job. The carpenter works 5 days a week and 8 hours each day. How many days are required to complete the job?

Step 1: What are you trying to determine?

Number of days needed to complete the job.

Step 2: What numbers are important?

112 hours for the total job, carpenter works 8 hours per day. (To answer the question, it doesn't matter that he works 5 days a week.)

Step 3: Which mathematical operation is needed to find the answer?

Division is required to find out how many days to complete the job.

Step 4: What is your equation?

$$112 \text{ h} \div 8 \text{ h/day} = ?$$

Completing the calculation tells us that the answer is 14 days.

Converting between Imperial and Metric Measurements

The metric system is used throughout the BC Building Code. Metric measurements are used in the drawings for the concrete portion of many large projects, and for most government projects. Ready-mix suppliers sell concrete by the cubic metre.

Imperial measurements are used by most carpenters and building material suppliers. Carpenters often need to convert metric measurements to imperial or imperial to metric.

Metric Units

The metric system has specific units of measure for a number of dimensions. Length (metre), mass (gram), and volume (metre, or litres for small to moderate liquid measures) are the most common basic units. The table below outlines some of the more common metric units of measure encountered in the carpentry trade.

DIMENSION MEASURED	BASE UNIT	SYMBOL
Length	metre	m
Mass	gram	g
Area	square metre	m ²
Volume	cubic metre or litre	m ³ ; L or l
Force	Newton	N
Pressure	Pascal	Pa
Temperature	degree Celsius	°C

Metric Prefix

The metric system is based on the number 10. Each level is 10 times greater than the level before it (10, 100, 1000, etc.), or 10 times smaller than the level above it ($\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$). Each level away from the base unit is assigned a prefix for verbal or written use. The most common prefixes are listed in the table below.

PREFIX	SYMBOL	VALUE
mega	M	1 000 000
kilo	k	1000
centi	c	0.01
milli	m	0.001

Combining prefixes with base units provides us with units that may be used to measure many things. For example, a kilogram (kg) is 1000 grams. A centimetre (cm) is $\frac{1}{100}$ of a metre. Some common metric measurements don't follow the prefix system above. For example, a "tonne" is a mass measurement equal to 1000 kg and its symbol is "t." It's helpful to have a mental "feel" or picture for each unit. For instance, how long is a metre? How heavy is a kilogram? What does a megaPascal of pressure feel like? If you know what a million Pascals feels like, it will help you understand what many MPas could do to your concrete forms.

Converting Within the Metric System

As the quantity of a dimension increases or decreases, add the appropriate prefix and change the number by moving its decimal place.

$$1000 \text{ Pa} = 1 \text{ kilopascal} = 1 \text{ kPa}$$

$$0.01 \text{ metres} = 1 \text{ centimetre} = 1 \text{ cm}$$

$$0.001 \text{ metres} = 1 \text{ millimetre} = 1 \text{ mm}$$

Converting Between Metric and Imperial Measurements

Since the *BC Building Code* is written using metric measurements and most carpenters work in imperial, it's important to be able to convert from metric to imperial. Examples of this include the rise, run, and required headroom for stairs, foundation heights and thicknesses, span tables, and almost all *Code* items. Site plans usually give elevations and property line distances in metric.

Imperial measurements must be converted to metric when ordering concrete.

Memorizing conversion charts is not practical, but you should be able to use them. Most of the work done by carpenters can be converted using four conversion factors:

25.4 mm = 1" (soft conversion is done using 25 mm = 1 in.)

305 mm = 1' (soft conversion is done using 300 mm = 1 ft.)

1 m = 3.28'

0.7646 m³ = 1 yd³

Another easy and somewhat foolproof conversion is to use a combination metric and imperial tape measure. This type of tape measure can be awkward to use on the job site for measuring and laying out, but is worth having in the toolbox for converting between metric and imperial.

For example, the maximum rise for residential stairs is 200 mm. In Figure 2, the 20 represents 20 cm or 200 mm. Reading up from the 20 metric mark is 7 ⁷/₈".



Figure 2 — Combination metric/imperial tape measure

Doing this same conversion mathematically would require the following steps:

$$200 \text{ mm} / 25.4 = 7.874"$$

$$7.874" = 7" + 0.874"$$

$$0.874" \times 16 = 13.98 \text{ therefore } 0.874" = 14/16" \text{ or } 7/8"$$

Giving an answer of 7 ⁷/₈".

DIMENSION	METRIC TO IMPERIAL	IMPERIAL TO METRIC
Length	1 km = 0.621 mi	1 mi. = 1.609 km
	1 m = 3.28 ft.	1 ft. = 0.305 m
	1 cm = 0.394 in.	1 in. = 2.54 cm
	1 mm = 0.039 in.	1 in. = 25.4 mm
Area	1 m ² = 10.76 ft. ²	1 ft. ² = 0.093 m ²
	1 cm ² = 0.16 in. ²	1 in. ² = 6.45 cm ²
Volume	1 m ³ = 1.308 yd. ³	1 yd. ³ = 0.7646 m ³
Mass	1 tonne = 2204.6 lb.	1000 lb. = 0.454 tonnes
	1 kg = 2.205 lb.	1 lb. = 0.454 kg
	1 g = 0.035 oz.	1 oz. = 28.035 g
Force	1 kPa = 20.885 psf	1 psf = 47.88 Pa = 0.048 kPa

Figure 3 — Metric/Imperial conversion factors

Conversion Process

Whether converting metric to Imperial or vice versa, the process is the same. Multiply the measurement you have by the appropriate conversion factor. Your answer will only be as accurate as the accuracy of the initial value and the conversion factor.

Convert 38 mm to inches:

$$38 \times 0.039 = 1.482$$

$$= 1.5 \text{ inches}$$

Convert 123 miles to kilometres:

$$123 \times 1.609 = 197.907$$

$$= 198 \text{ km}$$

Convert 12 lbs. to kilograms:

$$12 \times 0.454 = 5.448$$

$$= 5.4 \text{ kg}$$

Convert 15 grams to ounces:

$$15 \times 0.035 = 0.525$$

$$= 0.53 \text{ oz.}$$

Using Scientific Calculators

All calculators have the basic abilities to do addition, subtraction, multiplication, and division. When choosing a calculator for carpentry purposes, bigger is not always better. Often, having too many functions on a calculator becomes confusing and is generally a nuisance. It should have exponent and square root functions (covered in Learning Task 2), Pi (π), fraction to decimal conversion, fractional keypad, parenthesis, and the three basic trigonometry functions (SIN, COS, and TAN). It should also have a DMS (degrees, minutes, and seconds) button.

The most common mistakes made when using calculators is not following BEDMAS or the proper use of brackets. The second most common error is due to numbers being entered incorrectly. A good quality calculator will show both the equation and the answer so it can be confirmed that the equation was entered correctly.

As with the old adage, “measure twice, cut once”, it’s wise to enter the equation into the calculator and record your answer, then clear everything and do it again to see if it gives the same result.



Now complete Self Test 1 and check your answers.

Self Test 1

1. Division is the process of:

2. Exponents are used when:

3. BEDMAS stands for:

4. What is π ?

5. Solve the following word problem:

A sheet of plywood subfloor covers 32 sq. ft. and requires 43 screws for fastening. The floor area is 960 sq. ft. How many screws are required in total?

6. What does it mean to have a 4:12 roof slope?

7. Convert 30 467 mm to metres.

8. Convert 0.025 m to millimetres.

9. Convert 100 mm to inches.

10. Convert 12 yds.³ to cubic metres.

11. A snow load of 2 kPa is the same as how many pounds per square foot (psf)?

12. Use your calculator to solve the following equation:

$$x = (20 + 5)2 \times 4 - 2$$

LEARNING TASK 2

Use Trade Math

This Learning Task looks at how to apply and use the mathematical concepts presented in Learning Task 1. Carpenters typically work in feet and inches, a base 12 system. Inches are normally measured to the nearest 16th of an inch (base 16). Carpenters are routinely required to convert inches and fractions of an inch into decimals (base 10), and when given decimal portions of feet and inches to convert to inches and fractions of an inch.

Other carpentry math includes ratio and proportion, percentage, geometry, algebra, and occasionally, trigonometry.

Working with Fractions

Fractions are used to indicate a portion of a whole and are written as a whole number over a whole number. If it takes four hours to get halfway ($\frac{1}{2}$) through a job, it will take another four hours to complete it. If one quarter ($\frac{1}{4}$) of a cake is eaten, then three quarters ($\frac{3}{4}$) of the cake remain.

Numerator

The numerator is the number above the fraction line and indicates how many equal parts are present. In the fraction $\frac{3}{8}$, the 3 is the numerator and indicates that there are three equal parts of the whole unit present.

Denominator

The denominator is the number below the fraction line and indicates how many equal parts make up a complete unit. In the fraction $\frac{3}{8}$, the 8 represents that the whole unit has been divided into eight equal parts.

Terms

The numerator and denominator are called the “terms” of the fraction.

Proper Fraction

A proper fraction has a numerator that is smaller than the denominator. $\frac{1}{2}$, $\frac{15}{16}$, and $\frac{31}{64}$ are examples of proper fractions.

Improper Fraction

An improper fraction has a numerator as large or larger than the denominator. $\frac{6}{4}$, $\frac{8}{8}$, and $\frac{35}{32}$ are examples of improper fractions.

Mixed Number

A mixed number consists of a whole number (e.g., 1, 3, 12, 54) and a proper fraction (e.g., $\frac{1}{2}$, $\frac{5}{8}$, $\frac{2}{3}$). The following are all mixed numbers: $1\frac{1}{2}$, $3\frac{1}{4}$, $2\frac{5}{8}$.

Mixed numbers are more commonly written than improper fractions. For example, it's more common to write $2\frac{3}{4}$ than $\frac{11}{4}$ even though they're equivalent.

An improper fraction can be converted to a mixed number by dividing the numerator by the denominator with the remainder becoming the new numerator.

$$\frac{21}{4}$$

= 5 with a remainder of 1

$$= 5\frac{1}{4}$$



Some calculators have an “a b/c” button that allows you to enter either form of a fraction.



Figure 1 — “a b/c” button on calculator

Common Factor

A common factor is a number that will divide into both the numerator and the denominator, resulting in smaller whole number terms. Consider the fraction $\frac{4}{6}$. Since both 4 and 6 are divisible by 2, the fraction can be reduced to $\frac{2}{3}$. Both fractions have the same value, but $\frac{2}{3}$ uses the lowest possible terms, which is the preferred way to express fractions.

Note that in addition to reducing a fraction to lowest possible terms, you can also increase a fraction to larger terms. For example, given the fraction $\frac{4}{5}$, multiplying both terms by 3 results in $\frac{12}{15}$. Both fractions have the same value, but $\frac{12}{15}$ uses higher terms. The most common reason for doing this is when finding the “lowest common denominator” (which is described below).

Reciprocal

The reciprocal of a fraction is found by switching the values of the numerator and the denominator. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ (which is usually written as just 4).

The reciprocal is also called the “inverse” of the original fraction. Reciprocals are often used when dividing by fractions.

Lowest Common Denominator

The term “lowest common denominator” (LCD) applies to a group of fractions that are to be added or subtracted from each other. These fractions need to be expressed in higher terms to get a denominator that’s common to all. For example, adding or subtracting $\frac{2}{5}$ and $\frac{1}{3}$ requires both fractions be expressed with the lowest common denominator. The lowest common denominator of 5 and 3 is 15, so in this case, $\frac{2}{5}$ is converted to $\frac{6}{15}$, and $\frac{1}{3}$ to $\frac{5}{15}$. With the denominators the same, they can now be added or subtracted.

The more fractions involved, the harder it is to find the lowest common denominator. For example, the lowest common denominator of 2, 4, and 5 is 20. Note that the lowest common denominator might be one of the numbers itself. For example, the lowest common denominator of 2, 3, and 6 is 6.

Conditions for Adding and Subtracting Fractions

Only fractions with the same denominator (sometimes called “like” fractions) can be added or subtracted with each other. Fractions with different denominators (sometimes called “unlike” fractions) must be changed to higher terms to find the lowest common denominator.

Adding Fractions

When adding like fractions, simply add all the numerators together, and place that number over the common denominator.

$$\frac{1}{5} + \frac{2}{5} + \frac{3}{5} = \frac{6}{5}$$

Converting $\frac{6}{5}$ to a mixed number gives the final answer of $1\frac{1}{5}$.

In order to add unlike fractions, find the lowest common denominator, convert each fraction to the LCD, and add the numerators together.

$$\frac{2}{3} + \frac{1}{4} + \frac{1}{6}$$

The lowest common denominator of 3, 4, and 6 is 12, so convert the fractions to:

$$\frac{8}{12} + \frac{3}{12} + \frac{2}{12}$$

Now add the numerators together to get $\frac{13}{12}$.

Converting $\frac{13}{12}$ to a mixed number gives the final answer of $1\frac{1}{12}$.

Adding mixed numbers can be done in two steps: first add the fractions together and then add the whole numbers (including any whole number portion that results when adding the fractions). Combine the results to get the final answer.

$$2\frac{3}{4} + 4\frac{13}{16}$$

First, add the fractions only:

$$= \frac{3}{4} + \frac{13}{16}$$

$$= \frac{12}{16} + \frac{13}{16}$$

$$= \frac{25}{16}$$

$$= 1\frac{9}{16}$$

Now, add the whole numbers together (including the whole number 1 from the result above):

$$= 2 + 4 + 1$$

$$= 7$$

Finally, add the whole number portion (7) to the fractional portion ($\frac{9}{16}$) to get the final answer:

$$= 7\frac{9}{16}$$

Subtracting Fractions

When subtracting like fractions, simply subtract the numerators and place that number over the common denominator.

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

In order to subtract unlike fractions, find the lowest common denominator, convert each fraction to use the LCD, and then subtract the numerators.

$$\frac{7}{8} - \frac{5}{12}$$

The lowest common denominator of 8 and 12 is 24, so convert the fractions to:

$$\frac{21}{24} - \frac{10}{24}$$

Now subtract the numerators to get the final answer:

$$= \frac{11}{24}$$

In order to subtract mixed numbers, it's easiest if they are first converted to the lowest common denominator, and then each mixed number is converted to an improper fraction.

$$4\frac{1}{4} - 2\frac{1}{2}$$

Convert to the lowest common denominator:

$$= 4\frac{1}{4} - 2\frac{2}{4}$$

Convert each mixed number to an improper fraction:

$$= \frac{17}{4} - \frac{10}{4}$$

Subtract the numerators:

$$= \frac{7}{4}$$

Finally, convert the improper fraction into a mixed number:

$$= 1\frac{3}{4}$$

Multiplying and Dividing Fractions

Fractions can be multiplied or divided without having to find common denominators. The procedure for each process is outlined below.

Multiply Proper and Improper Fractions

Multiplying fractions is done by multiplying all the numerators and then multiplying all the denominators.

$$\frac{5}{8} \times \frac{2}{3}$$

Multiply the numerators (5×2), and the denominators (8×3) to get:

$$= \frac{10}{24}$$

Which can be reduced to the final answer of:

$$= \frac{5}{12}$$

Multiply Mixed Numbers

When multiplying mixed numbers, first convert to improper fractions and then perform the steps described above.

$$1\frac{1}{4} \times 2\frac{3}{8}$$

Convert both mixed numbers to improper fractions:

$$= \frac{5}{4} \times \frac{19}{8}$$

Multiply the numerators (5×19) and the denominators (4×8) to get:

$$= \frac{95}{32}$$

Convert back to a mixed number to get the final answer of:

$$= 2 \frac{31}{32}$$

Divide Proper and Improper Fractions

Dividing a number by a fraction is the same as multiplying by the reciprocal of the fraction. For example, 5 divided by $\frac{2}{3}$ is the same as 5 multiplied by $\frac{3}{2}$.

$$\frac{1}{4} \div \frac{2}{5}$$

The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$, so we can rewrite this equation as:

$$= \frac{1}{4} \times \frac{5}{2}$$

Using the procedure for multiplying fractions results in a final answer of $\frac{5}{8}$.

$$\frac{11}{4} \div \frac{1}{2}$$

The reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$, so we can rewrite this equation as:

$$= \frac{11}{4} \times \frac{2}{1}$$

Using the procedure for multiplying fractions gives:

$$= \frac{22}{4}$$

Converting to a mixed number results in:

$$= 5 \frac{2}{4}$$

Which when reduced to lowest terms, gives a final answer of:

$$= 5 \frac{1}{2}$$

Divide Mixed Numbers

In order to divide mixed numbers, first convert them to improper fractions and then perform the steps described above.

$$5 \frac{1}{3} \div 2 \frac{4}{9}$$

Converting to improper fractions gives:

$$= \frac{16}{3} \div \frac{22}{9}$$

The reciprocal of $\frac{22}{9}$ is $\frac{9}{22}$, so we can rewrite this equation as:

$$= \frac{16}{3} \times \frac{9}{22}$$

Using the procedure for multiplying fractions gives:

$$= \frac{144}{66}$$

Converting to a mixed number results in:

$$= 2 \frac{12}{66}$$

Which when reduced to lowest terms, results in a final answer of:

$$= 2 \frac{2}{11}$$

Concepts and Terms Involved with Decimals

The imperial system commonly uses fractions. However, stair construction, rafter lengths, and concrete volumes all use decimals. The metric system uses decimals. Carpenters need to understand the terms associated with decimals and have the ability to process mathematical equations.

Place Value

Place value is the value of each digit in a decimal. Consider the number 0.3. The 3 is in the “tenths” position or place and has a value of $\frac{3}{10}$ of a complete unit.

As an example, the number 1.246835 can be broken down into its place values as follows:

1	2	4	6	8	3	5
Whole numbers	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths	Millionths

The “2” is in the tenths place and has a value of $\frac{2}{10}$ of a complete unit. The “4” is the hundredths place and has a value of $\frac{4}{100}$ and so on. With the “5” in the millionths place, this number is accurate to the millionth of one complete unit.

When there is no whole number in the value, include a leading zero for clarity. That is, write 0.3579 rather than .3579 so that it's obvious what is meant.

Significant Digits

In Learning Task 1, a measurement was defined with three criteria: numbers, units, and degree of accuracy. Significant digits (figures) deal with the third criteria and are used to describe how accurate a measurement is.

Using significant digits helps focus on the numbers that are actually relevant. Significant digits are generally considered rules for rounding but they also ensure the answers arrived at are reasonable given the accuracy of the measurements.

Humans do not take perfect measurements—some degree of uncertainty always exists. If a carpenter says a desk is 2' 3 1/4", what he really means is that the desk is about 2' 3 1/4" long. It's more accurate for him to say that it's somewhere between 2' 3 3/16ths" and 2' 3 5/16ths". Generally, the last part of any measurement taken is "a judgment call." Whether you say 27 1/4" or 2' 3 1/4", the 1/4" part of the measurement is a best guess.

When in doubt, remember the idea of significant digits in measurement is 1/16 of an inch in the imperial system and one millimetre in the metric system. Both the 1/16th and the mm are best guesses, and the measurements (as well as any calculations) will never be more accurate than these.

Rules for Significant Digits

Significant digits are the digits that represent portions of a measurement that have actual value. For example, 0.035 m has two significant digits: the "3" and the "5." The zeroes have no value but position the 35/1000 of a metre in the correct place within the decimal. The significant digits are easier to see if the measurement is written as 35 mm.

Non-zero digits are always significant digits. Zero digits can be non-significant or significant depending on their position within the number. The following rules apply to zeroes positioned within a number:

- Zeroes placed before other digits are not significant: 0.035 has two significant digits.
- Zeroes placed between other digits are always significant: 30.04 has four significant digits.

Adding zeroes after a decimal digit indicates the degree of accuracy of the number. If you measured an elevation to the closest metre with a string level and found it to be 3 m in height, record your reading as "3 m." If you measure the same elevation with a builder's level and found it to be 3.000 m in height, record your reading as "3.000 m" to indicate its accuracy.

Rounding

Rounding is the process of reducing the number of significant digits in a value which will also reduce the precision of the value. A measurement may be too precise to be practical—it needs to be rounded to be useful. This often occurs when measurements are divided. Carpenters typically work to the nearest millimetre when in metric or to the nearest $\frac{1}{16}$ of an inch when working in imperial measurements.

Find the first quarter point of a beam that's 25' 3" long.

$$25' 3" \div 4 = 6.3125 \text{ feet}$$

Rounding to the hundredths place yields 6.31 feet (or 6' 3 $\frac{3}{4}$ ").

Rounding is accomplished by looking at the last digit and using its value to round up or round down. If the final digit is "4" or less, simply drop that digit. If the final digit is "5" or more, drop the digit and add one to the preceding digit. For example, 0.874 is rounded down to 0.87, whereas 0.876 is rounded up to 0.88.

Rounding up or down can be done for more than one place value at a time. To do this, consider only the digit one greater than the place value that is being rounded to, and perform the rounding as described above. For example, to round 3.72453 m to the nearest hundredths of a metre, consider the digit in the thousandths position. Since the thousandths digit is a 4, round down by simply dropping it (and all subsequent digits) to get 3.72 m.

Using the word problem example from Learning Task 1, recall that 14 days are needed to complete the job. You assign three carpenters to the job and they start on Monday. The clients want to know when they will have a finished project. What do you tell them?

$$14 \text{ days} \div 3 \text{ carpenters} = 4.6666... \text{ days.}$$

4.6666... days is equivalent to 4 days, 5 hrs, and 20 minutes but this is an unreasonable degree of accuracy. Instead, 4.6666... should be rounded to 5 days when informing the clients.

Converting Between Decimals and Fractions

Converting a fraction to a decimal is a simple task, whereas converting a decimal to a fraction takes more work.

Fraction to Decimal

Converting a fraction to a decimal is accomplished by dividing the numerator by the denominator. This works with both proper and improper fractions. Converting a mixed number requires that only the fraction portion of the mixed number is converted.

$$\frac{3}{8}$$

$$= 3 \div 8$$

$$= 0.375$$

$$\frac{15}{16}$$

$$= 15 \div 16$$

$$= 0.9375$$

$$2\frac{9}{11}$$

$$= 2 + (9 \div 11)$$

$$= 2.81$$

Decimal to Fraction

To change a decimal to a fraction, look at the place value of the last digit. Rewrite the digits above this place value, then reduce the fraction to its lowest terms.

$$0.5$$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

$$0.45$$

$$= \frac{45}{100}$$

$$= \frac{9}{20}$$

Since most tape measures are marked in $\frac{1}{16}$ ths, an answer of $\frac{9}{20}$ is not very useful. To find the answer to the closest 16th, the decimal is multiplied by 16 and the answer represents the closest 16th of an inch.

$$\begin{aligned} &0.45 \\ &= 0.45 \times 16 \\ &= 7.2 \end{aligned}$$

Therefore 0.45 is closest to $\frac{7}{16}$.

Add, Subtract, Multiply, and Divide Decimals

Using any of the four basic mathematical operations on decimals is the same as with whole numbers. With a calculator, simply input the numbers with the decimal, select the math operation to be performed, then finalize your calculation with the equal sign. If doing the calculations on paper, make sure to keep the place values aligned for all the numbers.

Adding Decimals

Add 3.75, 67.005, 892.2, 0.5325, and 326.

Align the decimal points to keep all the place values aligned, add each column from right to left, carrying groups of 10 as required.

$$\begin{array}{r} 3.75 \\ 67.0050 \\ 892.2000 \\ 0.5325 \\ \underline{326.0000} \\ 1289.4875 \end{array}$$

Subtracting Decimals

Align the decimal points again and add zeroes to make it easier to move units of 10 from one place value to a lower place value.

Subtract 0.0125 from 3.25.

$$\begin{array}{r} 3.2500 \\ - 0.0125 \\ \hline 3.2375 \end{array}$$

Multiplying Decimals

When multiplying decimals by hand, first count the total number of decimal places in both numbers. Then multiply the numbers as though they are whole numbers (that is, ignore the decimal places entirely). Once you have an answer, place the decimal point the same number of places as the total of the original numbers.

Example

Multiply 8.08×0.625 .

8.08 has **two** decimal places, and 0.625 has **three** decimal places, for a total of **five** decimal places.

Multiply 808×0625 :

$$\begin{array}{r} 0625 \\ \times 808 \\ \hline 505000 \end{array}$$

Place the decimal point so that there are **five** decimal places (the same as the total in the original numbers):

5.05000

Trailing zeroes can be dropped to give a final answer of 5.05.

Dividing Decimals

When dividing decimals by hand, move the decimal point of both numbers an equal number of spaces to the right until both are whole numbers. (If the numbers do not have the same number of decimal places, add as many zeroes as needed.)

Divide 481.92 by 0.024.

Move the decimal point of both numbers three places to the right: 481.92 becomes 481920, and 0.024 becomes 24.

Divide the resulting whole numbers:

$$481\,920 \div 24 = 20\,080$$

Note that you do not move the decimal places “back” after completing the division, so the final answer is 20 080.

Ratio and Proportion

Ratio and proportion are used by carpenters in several ways. Common ratio specifications are included in all scaled drawings. Ratio and proportion can be used when working with similar triangles. Ground, sidewalk, roof, and deck slopes are given as ratios, along with many other *BC Building Code* and WorkSafeBC requirements. Mixing fuel for a two-cycle chainsaw requires a fuel mixture of 50 parts gas to 1 part oil, and this is a ratio 50:1.

Ratio

Ratio is a comparison between two related quantities. Ratios use quantities that have the same unit of measure (e.g., metres to metres, inches to inches, or litres to litres).

Ratios can be expressed in several ways:

- 1 to 2
- $\frac{1}{2}$
- 1:2
- $1 \div 2$

To avoid confusion, how the two quantities are being compared must be stated clearly. For example, a ground slope could be expressed as 1:3 or 3:1 depending on whether the vertical is being compared to the horizontal or vice-versa. To avoid this confusion, state it as 1:3 (V:H), which means that the slope goes up 1 for every horizontal distance of 3.

Note that unlike fractions, ratios are not limited to using only whole numbers for the first term (e.g., 1.2:3 is a valid ratio).

If you need to express a ratio in the form “x:1,” simply divide the first term by the second and replace the “x” with the result.

Express 7:2 in the form “x:1.”

$$7 \div 2 = 3.5 \text{ therefore the ratio can be expressed as: } 3.5:1$$

Proportion

Ratios are “proportionate” when they are equal to each other. For example, the ratios 1:2 and 2:4 are equal, so they are said to be proportionate or “in proportion” to each other. Cross-multiplication can be used to determine whether two ratios are proportionate or not. (To “cross-multiply” fractions, multiply the numerator of the first fraction with the denominator of the second. Then multiply the denominator of the first with the numerator of the second.)

Are the ratios 2:1 and 6:3 proportionate?

First, write the ratios as fractions:

$$\frac{2}{1} \quad \frac{6}{3}$$

Now, cross-multiply to get the following two equations:

$$2 \times 3 = 6$$

$$1 \times 6 = 6$$

Since the value of both equations is the same, the ratios are in proportion to each other.

This principle can be used to determine an unknown value if given the other three values.

Example

A drawing has a scale (or ratio) of 1:100. That is, 1 cm on the drawing is equal to 100 cm (1 metre) in real life.

A beam span on the drawing measures 4.5 cm long. How long is the beam span in real life?

The two ratios are: $\frac{1}{100} = \frac{4.5}{bs}$ (where bs is the length of the beam span in real life).

Using cross-multiplication, we get the following equation:

$$1 \times bs = 100 \times 4.5$$

Therefore $bs = 450$ and the beam span is 450 cm (4.5 m) long in real life.

Percentage

Percentage is the numerator of a fraction when the denominator is 100. For example, $\frac{25}{100}$ is the same as 25 percent (25%). Percentages are used to compare something with another value. For example, if 75% of a pile of bricks have been used, this is comparing the number of bricks used with the total number overall.

Percentages decrease or increase the original amount. They may be used to plan for wastage when ordering materials, for adding taxes, to plan for contingencies, to allow for overhead, or for profit when bidding on a job. They can also be used to report statistics, such as 86% of the class passed the Week 2 test.

Since 100% equals $\frac{100}{100}$ (or 1), if the cost of lumber was unchanged from last week, it could be said that lumber was selling for 100% of last week's price. If the cost had doubled since last year, it can be said that lumber was selling for 200% of last year's cost, or it could be said that lumber prices had gone up by 100%.

Finding the Percent Change

Division is used to find the percent change in the amount or cost of something.

Example

If 10-foot-long 2×4 lumber was selling for \$3.44 each, and the price went up to \$4.00, that's an increase of \$0.56:

$$4.00 - 3.44 = 0.56$$

The difference (0.56) is then divided by the original amount (3.44) to get:

$$\frac{0.56}{3.44} = 0.16 \text{ or } 16\%$$

The price of lumber has gone up by 16%.

This method works whether the change is an increase or decrease of the original number.

Finding the Percent Amount

To find the value of a percentage change to an item, change the percent to a decimal and multiply the original amount. (To change a percentage to a decimal, divide it by 100. For example, $12\% = \frac{12}{100} = 0.12$.)

Example

10-foot-long 2×4s are selling for \$3.44 but 12% needs to be added for GST and PST. How much tax is charged per 2×4?

$$\begin{aligned} \$3.44 \times 0.12 &= \$0.4128 \\ &= \$0.41 \end{aligned}$$

How much tax per 2×4 is there if only the 5% GST is charged?

$$\begin{aligned} \$3.44 \times 0.05 &= \$0.172 \\ &= \$0.17 \end{aligned}$$

To get the total cost of each 2×4 with the 12% tax included, add 1 to the percentage decimal (i.e., multiply by 1.12 instead of 0.12).

$$\begin{aligned} \$3.44 \times 1.12 &= \$3.8528 \\ &= \$3.85 \end{aligned}$$

Working with Equations and Formulas

As covered in Learning Task 1, simple equations and formulae are used to solve math problems. Learning to manipulate equations to determine an unknown quantity is relatively easy as long as fundamental rules are followed.

An equation is a statement of equality between two quantities and is indicated by the equal sign (=). All equations must be true. An equation will usually have one unknown whereas a formula will have several.

Since both quantities (i.e., both sides) of an equation are equal, they will stay equal if both sides are manipulated identically. In other words, you can add, subtract, multiply, divide, square, or take the square root of both sides as long as you do it equally to both sides.

Solve the following equation (i.e., determine the value of "A"):

$$5 + \frac{A}{9} = 2304$$

Step 1: Subtract 5 from both sides:

$$5 + \frac{A}{9} - 5 = 2304 - 5$$

$$\frac{A}{9} = 2299$$

Step 2: Multiply both sides by 9:

$$9 \times \frac{A}{9} = 9 \times 2299$$

$$A = 20691$$

Geometry

Geometry is a branch of mathematics that deals with size, space, and the relative position of shapes. Carpenters mainly need to find perimeters, areas, and volumes and work with triangles, circles, and polygons.

Shapes

The basic shapes that you'll work with include rectangles, squares, trapezoids, triangles, and circles. Although not shapes, angles and straight lines are also important when working with geometry.

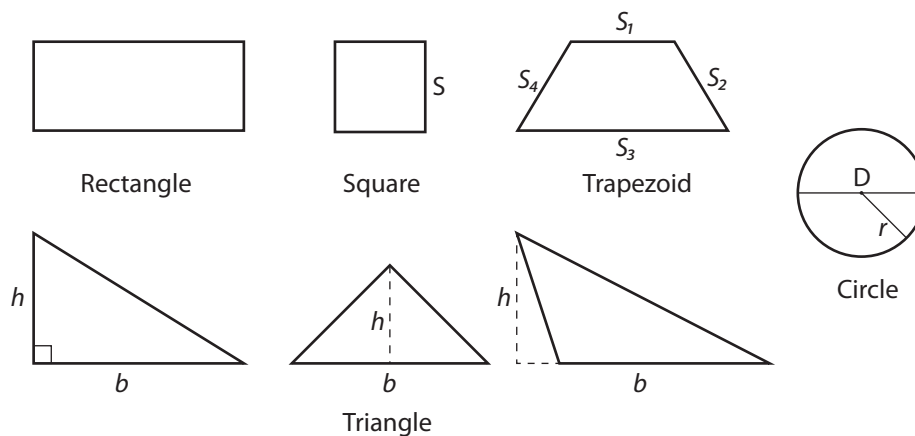


Figure 2 — Five geometric shapes

Rectangle

A rectangle is a four-sided shape with four 90° angles. Because of the angles, the opposite sides are parallel and equal in length.

Rectangles are used to lay out excavations and foundations. The two diagonals of a rectangle are always equal. If the opposite sides are equal in length and the diagonals are equal, then each corner is 90 degrees.

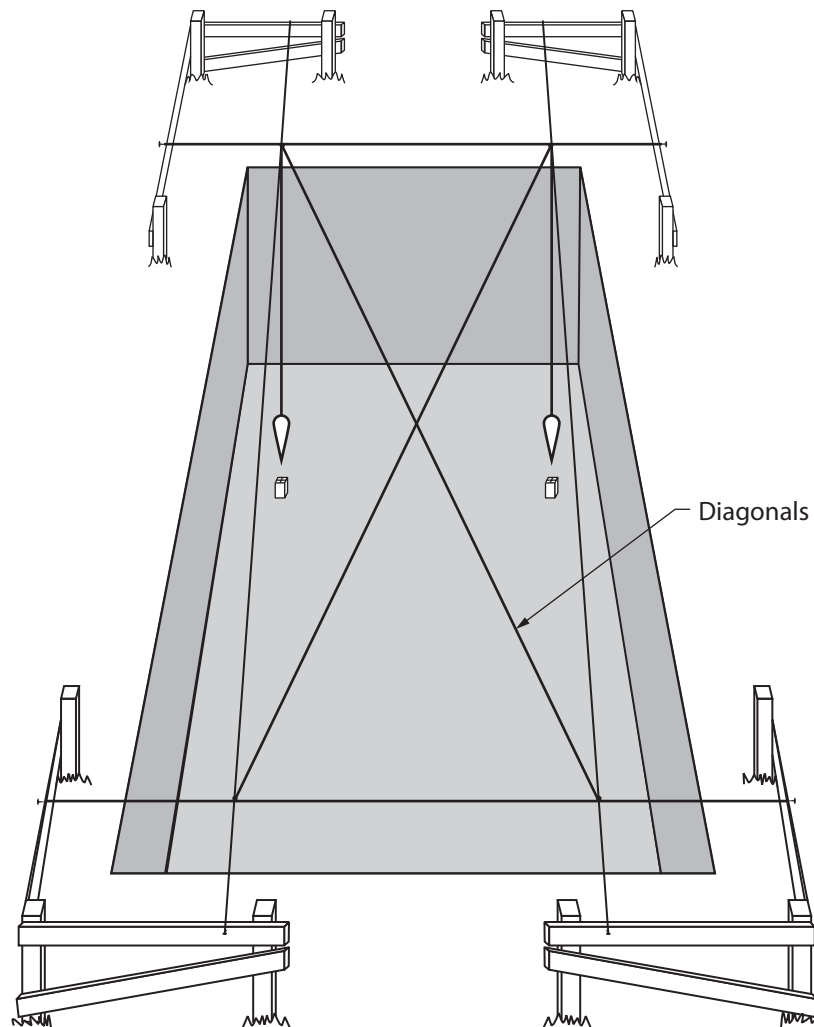


Figure 3 — Using batter boards to lay out a rectangle – opposite sides must be equal and the diagonals must also be equal

Square

A square is a rectangle which has all four sides equal in length.

Trapezoid

A trapezoid is another four-sided shape that has only one pair of parallel opposite sides. It resembles a rectangle with a triangle stuck on one or both ends.

Triangle

A triangle is a three-sided shape.

Circle

A circle is defined as all the points in a plane that are an equal distance from the single centre point. This distance is known as the circle's radius. Doubling the radius gives the circle's diameter.

Angles and Lines

An angle is formed by two intersecting lines. It's measured by the degrees of rotation a line sweeps through from a fixed point. Knowledge of the terms and concepts associated with angles will help you understand how geometric rules are applied for carpentry purposes.

Degree

Degree is the common unit of measure of angles and its symbol is: $^{\circ}$.

A complete rotation consists of 360° . An angle that is $\frac{1}{4}$ of a complete rotation is a 90° angle (and is commonly called a "right angle").

Vertex

The vertex is the point about which a line rotates to form an angle. In the angle ABC in Figure 4, B is the vertex.

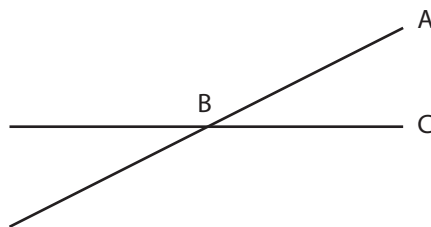


Figure 4 — Vertex

Angle Types

Angles are classified by their magnitude (that is, how wide or narrow they are). There are several common units for measuring angles: degrees, radians, and gradians. The most common (and the one used in this Learning Task) is degrees. (When using a calculator, make sure it's in "degree mode"—the "DRG" key is used to toggle between degrees, radians, and gradians.)

Right Angles

An angle that forms exactly 90° is known as a right angle. Right angles are often identified by a square symbol placed at the vertex.



Figure 5 — Right angle

Acute Angles

An angle less than 90° is known as an acute angle.

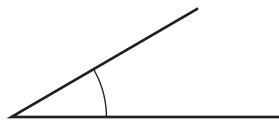


Figure 6 — Acute angle

Obtuse Angles

Obtuse angles have a measurement greater than 90° but less than 180° . Sometimes an arc is placed at the vertex to indicate which angle is being measured.

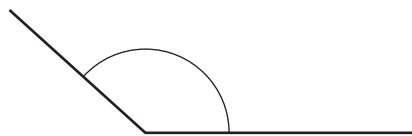


Figure 7 — Obtuse angle

Straight Angles

An angle that is exactly 180° is known as a straight angle. A straight angle is a line, which is defined as the shortest distance between two points.



Figure 8 — Straight angle

Complementary Angles

Complementary angles are two angles which add up to exactly 90° .

Angles ABC and CBD are complementary because together they equal exactly 90° .

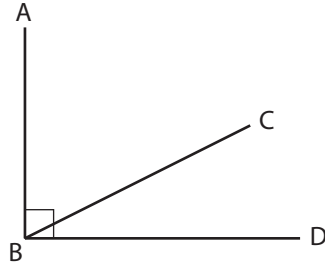


Figure 9 — Complementary angles

Supplementary Angles

Supplementary angles are two angles that add up to exactly 180° .

Angles ABC and ABD are supplementary because together they equal exactly 180° .

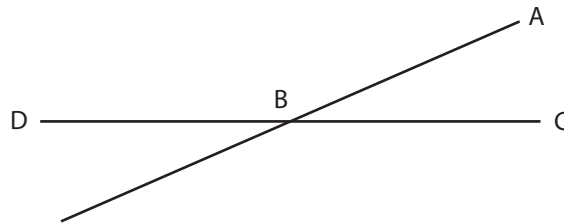


Figure 10 — Supplementary angles

Opposite Angles

Two intersecting lines will form four separate angles. For example, in Figure 11, angles A, B, C, and D are formed.

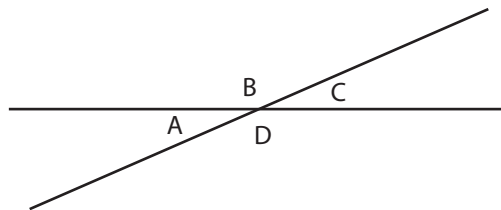


Figure 11 — Intersecting lines form four angles

Angles A and C are opposite, as are angles B and D. Opposite angles have the same magnitude.

Angles and Parallel Lines

Two straight lines are parallel when they run in exactly the same direction. Floor joists are usually laid out to run parallel, as are studs and rafters. Figure 12 shows a stud wall with a diagonal brace. Three of the many angles formed are labelled.

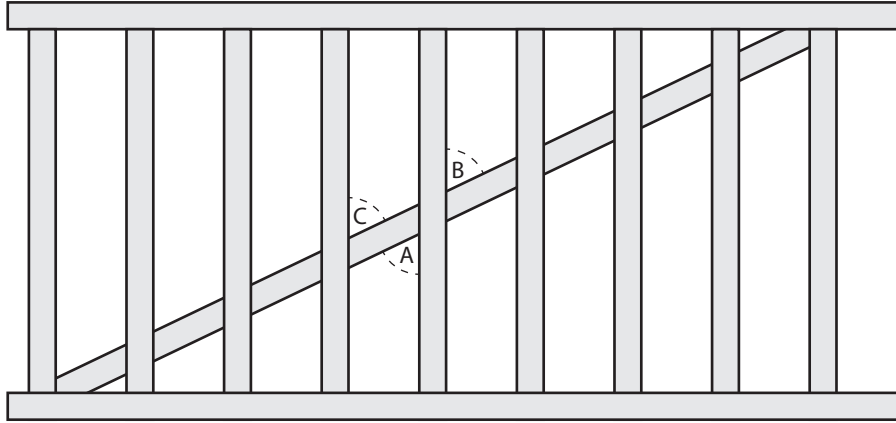


Figure 12 — Stud wall with diagonal brace

Angles A and B are opposite, so they are equal. Angles B and C are equal as they are formed by the same straight line intersecting parallel lines.

Units of Angle Measurement

A complete rotation contains 360 degrees. Each degree is further divided into 60 minutes, and each minute is further divided into 60 seconds. (The terms “minutes” and “seconds” have nothing to do with time.)

The symbol for minutes is: ‘

$14^{\circ} 03'$ (14 degrees, 3 minutes)

The symbol for seconds is: “

$10^{\circ} 40' 20''$ (10 degrees, 40 minutes, 20 seconds)

Minutes and seconds should always be written using two digits. (e.g., if a measurement is 3 minutes, write 03' rather than 3'.)

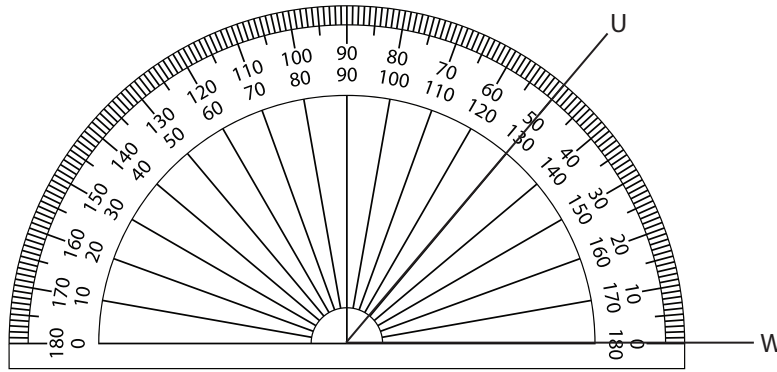


Figure 13 — Using a protractor to measure the UVW angle of 50 degrees.
(This protractor is not precise enough to measure minutes or seconds.)

On land development plans, rotations are often measured in degrees, minutes, and seconds (e.g., $15^{\circ} 35' 40''$). In other documents, angles may be measured in decimal degrees (e.g., 34.584°).

Working with Triangles

A triangle is a three-sided shape. It has three angles and they always add up to 180° .

Right Triangle

A triangle containing a right angle (90°) is known as a “right triangle.” The sum of the other two angles must be 90° .

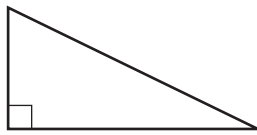


Figure 14 — Right triangle

Equilateral Triangle

If all three sides of a triangle are the same length, then it’s an equilateral triangle. Each of the angles in an equilateral triangle is 60° .

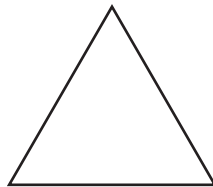


Figure 15 — Equilateral triangle

Isosceles Triangle

An isosceles triangle is one where two sides are equal in length but the third is not. The two angles at the base of an isosceles triangle are equal.

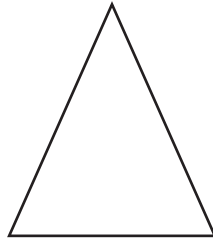


Figure 16 — Isosceles triangle

Similar Triangles

Two triangles which have angles of the same magnitude but different length sides are known as “similar triangles.”

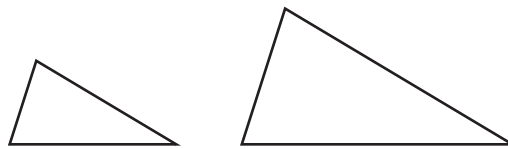


Figure 17 — Similar triangles

The 3:4:5 Triangle

A 3:4:5 triangle is a triangle that has sides of length 3, 4, and 5. It’s special because it will form a right triangle. The side opposite the right angle is called the hypotenuse and is designated by the letter “c.” The vertical leg of the triangle is designated “a” (altitude) and “b” is used to represent the base of the triangle.

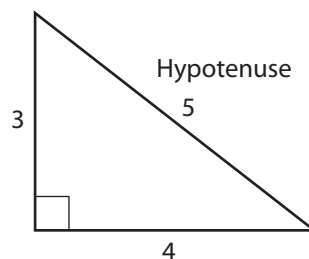


Figure 18 — 3:4:5 triangle

The fact that a 3:4:5 triangle must form a right angle is very useful for carpenters:

1. Create a mark 3 units along one component. (The units can be any measure you choose: centimetres, inches, feet, metres, etc.)
2. Create a mark 4 units along the other component.
3. If the distance between the two marks is 5 units, then the components are at right angles to each other.

Since a 3:4:5 triangle is a similar triangle to a 6:8:10 and a 9:12:15 triangle, they or any multiple of 3:4:5 can be used for this purpose. Higher multiples are used to improve accuracy.

Pythagorean Theorem

The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Written as a formula it is: $a^2 + b^2 = c^2$.

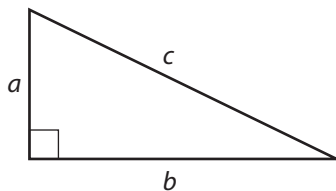
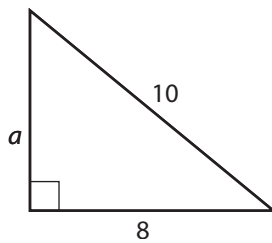


Figure 19 — Sides of a right triangle

This theorem can be used to find an unknown value in a right triangle.

Find “a” in the following right triangle:



$$a^2 + b^2 = c^2$$

$$a^2 + 8^2 = 10^2$$

$$a^2 + 64 = 100$$

$$a^2 = 100 - 64$$

$$a^2 = 36$$

$$a = 6$$

Finding Perimeter Distances

Calculating the perimeter (the distance around an object) involves a formula which is dependent on the shape of the object.

Perimeter Formula

The perimeter formulae related to common objects are listed in Figure 20. A complex shape that doesn't match the standard shapes can be divided into smaller standard shapes.

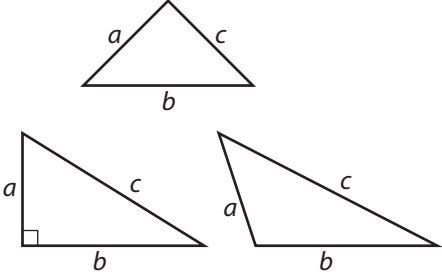
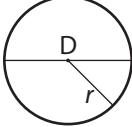
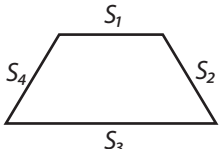
 Rectangle	$P = 2L + 2W$ or $P = 2(L + W)$ Where L = length W = width
 Triangle	$P = a + b + c$ Where a = the length of a side b = the length of a side c = the length of a side
 Square	$P = 4S$ Where S = the length of a side
 Circle	$C = \pi D$ or $C = 2\pi r$ Where C = circumference (perimeter) D = diameter r = radius $\pi = 3.1416$
 Trapezoid	$P = S_1 + S_2 + S_3 + S_4$ Where S = the length of a side

Figure 20 — Perimeter formula

A square has sides that measure 25 cm. What is its perimeter?

$$P = 4S$$

$$P = (4)(25 \text{ cm})$$

$$P = 100 \text{ cm}$$

What is the circumference (perimeter) of a circle with a diameter of 55 mm?

$$C = \pi D$$

$$C = \pi (55 \text{ mm})$$

$$C = 172.8 \text{ mm}$$

Finding Areas

Calculating the area of an object involves specific formulae based on its shape. The area of complex shapes can be calculated by dividing the shape into simpler forms, calculating their areas, and then adding to get the grand total.

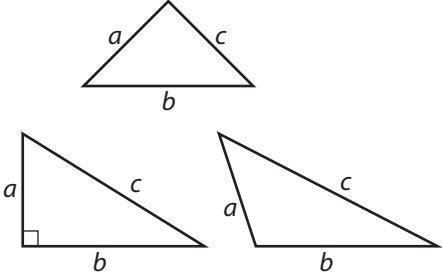
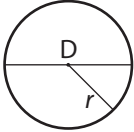
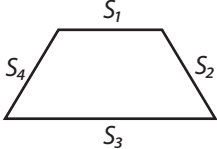
 <p>Rectangle</p>	$A = LW$ Where L = length W = width
 <p>Triangle</p>	$A = \frac{1}{2}bh$ Where b = base h = height
 <p>Square</p>	$A = S^2$ Where S = the length of a side
 <p>Circle</p>	$A = \pi r^2$ or $A = 0.79 D^2$ Where D = diameter r = radius $\pi = 3.1416$
 <p>Trapezoid</p>	$A = h \frac{S_1 + S_3}{2}$ Where h = height or altitude Where S = the length of a side

Figure 21 — Area formulae

A square has 25 cm sides. What is its area?

$$A = S^2$$

$$A = 25^2$$

$$A = 625 \text{ cm}^2$$

What is the area of a circle with a radius of 55 mm?

$$A = \pi \times r^2$$

$$A = \pi \times 55^2$$

$$A = 9503.3177$$

Since the radius is given in two significant digits (55 mm), the answer should be rounded to the same level of accuracy:

$$A = 9503 \text{ mm}^2$$

Finding Volume

Volume is defined as the amount of space occupied by an object. The term “volume” is often used to mean “capacity” or the number of cubic units enclosed within a container (such as a bin or tank).

Volume Formulae

The formulae for finding the volume of common objects are listed in Figure 22.

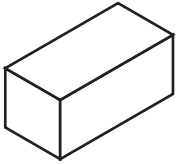
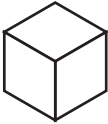
 Rectangular solid	$V = LWH$ Where L = length W = width H = height
 Cube	$V = S^3$ Where S = the length of a side
 Cylinder	$V = \pi r^2 h$ Where h = height r = radius $\pi = 3.1416$
 Sphere	$V = \frac{4}{3} r^3$ Where r = radius

Figure 22 — Volume formulae

Example

A rectangular water tank measures 12 m long by 5 m wide by 7.5 m high. How much water can the tank hold?

$$V = LWH$$

$$V = (12 \text{ m})(5 \text{ m})(7.5 \text{ m})$$

$$V = 450 \text{ m}^3$$

Example

A round concrete column (cylinder) is 0.800 m in diameter and 3.000 m high. What is the volume of concrete in the column? (Remember: a circle's radius is half its diameter.)

$$V = \pi r^2 h$$

$$V = \pi(0.400)^2(3.000)$$

$$V = \pi(0.16)(3.000)$$

$$V = 1.508 \text{ m}^3$$



Now complete Self Test 2 and check your answers.

Self Test 2

Note: This Self Test includes many questions for you to practise. Check with your instructor to see which questions are assigned.

1. Reduce $\frac{24}{64}$ to lowest terms.

2. Which fraction is larger: $\frac{3}{8}$ or $\frac{1}{32}$?

3. Express $\frac{83}{16}$ as a mixed number.

4. What is the reciprocal of $\frac{15}{16}$?

5. Express the following group of fractions with their lowest common denominator: $\frac{1}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{9}{12}$.

6. Add the following:

$$\frac{1}{2} + \frac{3}{4} + \frac{3}{16} =$$

7. Subtract the following:

$$6\frac{15}{16} - 3\frac{3}{8} =$$

8. Solve the following:

$$1\frac{1}{4} \times 8\frac{1}{2} =$$

9. Round 67.1275 to the nearest:

a. thousandth

b. hundredth

c. tenth

10. Round to the nearest thousandths:

a. 0.05072

b. 4.00009

11. Convert $\frac{5}{8}$ to a decimal.

12. Convert $1\frac{3}{16}$ to a decimal.

13. Express 2 as a decimal. Round to the thousandth place.

14. Change 0.06 to a fraction in 16ths.

15. What is 0.875 as a fraction in 16ths?

16. Complete the calculations as indicated:

a. $477.075 + 64 + 709.999 + 0.0025$

b. $37.2 - 36.864$

c. 0.25×0.375

d. $0.17 \div 0.0034$

17. A 2×4 is cut into five pieces. If the pieces are 35.62 cm, 49.225 cm, 23.5 cm, 78.43 cm, and 55.1 cm long, what was the original length of the 2×4? (Do not deduct for saw kerf thickness.)

18. 25.2 m = _____ in feet and inches

19. Solve for the unknown:

a. $3 \times 8 = Q \div 16$

b. $13 + 3 = 3 \times W$

c. $r^2 = \frac{45}{3.14}$

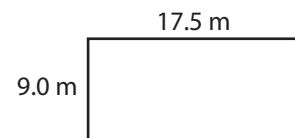
20. A journeyman carpenter makes \$32.00 an hour. A second-year apprentice makes \$24.00 an hour. What is the ratio in lowest terms of the journeyman's wage to the apprentice's wage?

21. A pulley with a diameter of 10 cm is driving a pulley that has a diameter of 6 cm. What is their ratio?

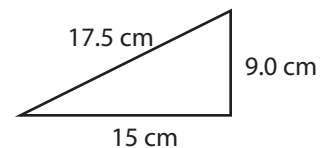
22. A steel pipe has a mass of 0.425 kg for every 0.33 m. What is the mass of 3.6 m of this pipe?

23. You're mixing boat gasoline and oil at a ratio of 50:1. You need 30 litres of fuel for the day's fishing trip. How much oil do you need to add?

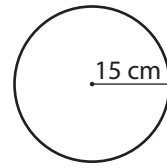
24. Find the perimeter of the following rectangle:



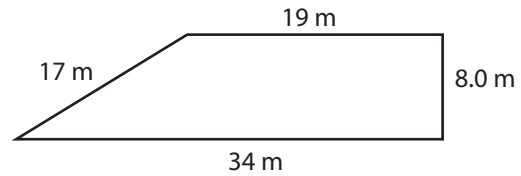
25. What is the perimeter of the following triangle?



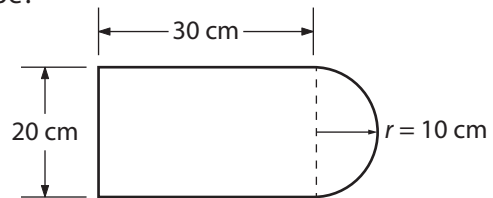
26. Find the circumference of the following circle:



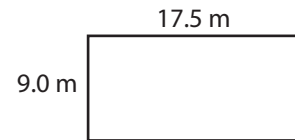
27. What is the perimeter of the following trapezoid?



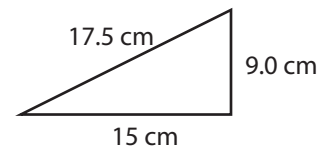
28. What is the perimeter of the following complex shape?



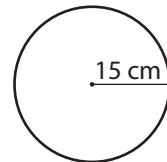
29. Find the area of the following rectangle:



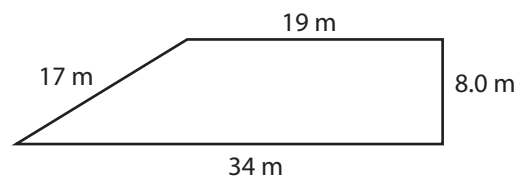
30. What is the area of the following triangle?



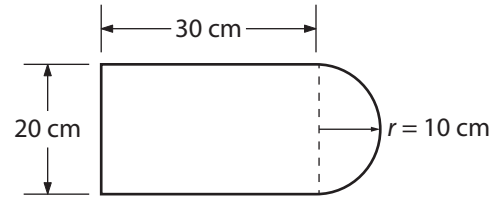
31. Find the area of the following circle:



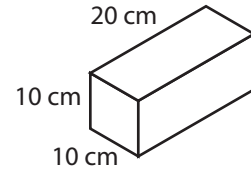
32. What is the area of the following trapezoid?



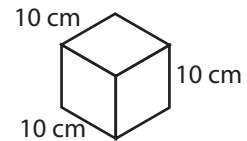
33. What is the area of the following complex shape?



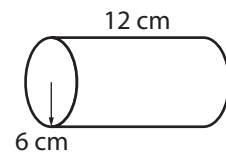
34. Find the capacity of the following rectangular tank:



35. What is the volume of the following cube?



36. Find the volume of the following cylinder.



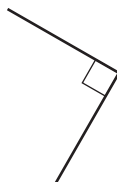
37. A cylindrical tank is 25 m long and 3 m in diameter. The tank is located inside a containment field that measures 30 m long by 5 m wide by 1 m high. If the tank develops a leak, is there enough volume in the containment field to hold the full tank capacity?

38. Identify the following angles:

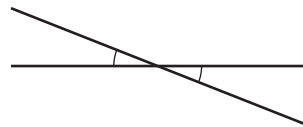
a.



b.

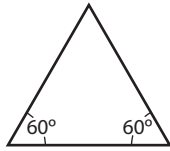


c.

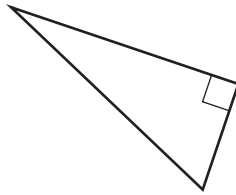


39. Identify the following triangles:

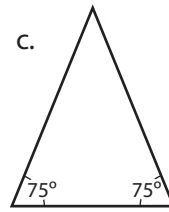
a.



b.

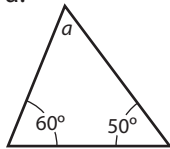


c.

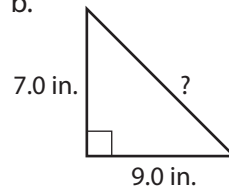


40. Calculate the missing measurements in the following triangles.

a.



b.



Answer Key

Self Test 1

1. separating a collection of objects into a number of equal groups
2. multiplying the same number multiple times
3. brackets, then exponents, then division and multiplication, and lastly addition and subtraction
4. Pi (3.1415...)
5. $x = (960 \div 32) \times 43$
 $x = 30 \times 43$
 $x = 1290$ screws
6. The roof slopes up at a ratio of 4 units for every run of 12 units (normally 4" for every 12" of run).
7. 30.467 m
8. 25 mm
9. $x = 100/25.4$
 $x = 3.937"$
 $x = 3 \frac{15}{16}"$
10. $x = 12 \times 0.7646$
 $x = 9.175 \text{ m}^3$
11. $x = 2 \times 20.885$
 $x = 41.77 \text{ psf}$
12. $x = (20 + 5)^2 \times 4 - 2$
 $x = (25)^2 \times 4 - 2$
 $x = 2500 - 2$
 $x = 2498$

Self Test 2

1. $\frac{3}{8}$
2. $\frac{1}{32}$
3. $5 \frac{3}{16}$
4. $\frac{16}{15}$
5. $\frac{6}{24}$, $\frac{20}{24}$, $\frac{9}{24}$, and $\frac{18}{24}$
6. $1 \frac{7}{16}$

7. $3\frac{5}{16}$
8. $10\frac{5}{8}$
9. a. 67.128
b. 67.13
c. 67.1
10. a. 0.051
b. 4.000
11. 0.625
12. 0.8125
13. 2.172
14. $\frac{1}{16}$
15. $\frac{7}{8}$
16. a. 1251.0765
b. 0.336
c. 0.09375
d. 50
17. 241.875 cm
18. 82.656' or 82' 7 $\frac{7}{8}$ "
19. a. $Q = 384$
b. $W = 5.333\ldots$
c. $r = 3.786$
20. 4:3
21. 10:6
22. 4.636 kg
23. 0.6 litres
24. 53 m
25. 41.5 cm
26. 94.25 cm
27. 78 m

- 28. 111.42 cm
- 29. 157.5 m^2
- 30. 67.5 cm^2
- 31. 706.858 cm^2
- 32. 212 m^2
- 33. 757.08 cm^2
- 34. 2000 cm^3
- 35. 1000 cm^3
- 36. 1357.168 cm^3
- 37. No (tank volume = 176.715 m^3 , field volume = 150 m^3)
- 38.
 - a. obtuse
 - b. right (90°)
 - c. opposite
- 39.
 - a. equilateral
 - b. right
 - c. isosceles
- 40.
 - a. 70°
 - b. 11.402 in.



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ISBN 978-0-7726-7030-4



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